Problems, March 2008

Problem 1. Find integers a, b, c, d, and n such that

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{6}} = \frac{a+b\sqrt{2}+c\sqrt{3}+d\sqrt{6}}{n}.$$

Problem 2. There are 20 people in a tango class. How many ways are there to divide them up into 10 dance couples? (This is 21st century Canada, there should be no assumptions based on sexual identity.)

Problem 3. A collection of 6 different integers is chosen at random from the set $\{1, 2, 3, \ldots, 48, 49\}$, with all choices equally likely. What is the probability that no 2 of these 6 numbers are consecutive?

Problem 4. Calculate $\displaystyle\sum_{k=1}^{\infty} \frac{k}{k^4+k^2+1}$.

Problem 5. Some non-negative integers can be expressed as the sum of two perfect squares. For example, $0 = 0^2+0^2$, $1 = 0^2+1^2$, $2 = 1^2+1^2$, $4 = 0^2+2^2$, and $5 = 1^2+2^2$. Some, like 25, 50, 65, and others, can even be so expressed in more than one way. But for example 3, 6, and 7 cannot be expressed as a sum of two squares. Among the integers from 0 to 999999, which ones are more common, the ones that can be expressed as the sum of two perfect squares, or the ones that cannot be so expressed?

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