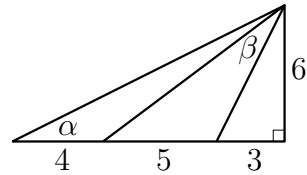


Problems, March 2009

Problem 1. In the right-angled triangle below, dimensions are as shown. Prove that $\alpha = \beta$.



Problem 2. (Please note that the originally posted version had two typos.) Let $f(x) = ax^2 + bx + c$, where a , b , and c are real numbers. Define $f'(x)$ by $f'(x) = 2ax + b$. Show that if $|f(x)| \leq 1$ on the interval $[-1, 1]$, then $|f'(x)| \leq 4$ on the same interval. No calculus please!

Problem 3. Show that $n^{n-1} - 1$ is divisible by $(n - 1)^2$ for every positive integer n .

Problem 4. How many ordered triples (x, y, z) of integers are there such that $|x| + |y| + |z| \leq 1000$?