Problems, March 2010

Problem 1. Let a be the number of digits in the decimal expansion of 2^{2010} and let b be the number of digits in the decimal expansion of 5^{2010}. What is the value of $a + b$?

Problem 2. For any positive integer n, let

$$F(n) = \frac{n^2}{(1.0005)^n}.$$

At what positive integer(s) n does $F(n)$ attain its maximum value? No calculus please!

Problem 3. Find all integers x such that $x^2 - 9x + 9$ is a perfect square. (A proof is needed that we have found them all.)

Problem 4. Among all triangles inscribed in the ellipse that has equation $x^2 + 4y^2 = 4$, what is the maximum possible area? And how many inscribed triangles are there with that area?