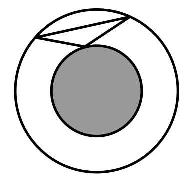
Problems, March 2011

Problem 1. Is there a perfect cube the sum of whose digits is 2011?

Problem 2. The outer circle in the picture has radius a, the inner circle has radius b, and the circles have the same centre. What is the maximum possible area of a triangle two of whose vertices are on the outer circle, with the third vertex on the inner circle, given that the entire triangle lies in the annulus between the two circles?



Problem 3. For which non-negative integers n is $2011^n > n^{2011}$?

Problem 4. Find a point (u, v) on the ellipse with equation $x^2 + 2y^2 = 1$ such that u and v are rational, and each, when expressed as a reduced fraction, has a denominator greater than 1000. Hint: Consider the line with slope m that passes through the point $(-1, 0)$.