Problems, March 2012

Problem 1. A five-digit number is formed by choosing independently and at random at random digits from $\{1, 2, 3, 4, 5\}$. Repeated digits are allowed; for example, the number might be 52534. What is the probability that the number is divisible by 6?

Problem 2. A super-kangaroo can jump any integer number of metres (forward direction only). In how many different ways can the kangaroo reach a point that is 50 metres from its starting point? A jump of 4 then 42 then 4 is different from a jump of 4 then 4 then 42.

Problem 3. Evaluate

$$\sum_{n=2}^{\infty} \frac{1}{n(n-1)^3},$$

given Euler's beautiful result that

$$1 + \frac{1}{2^2} + \frac{1}{3^3} + \cdots + \frac{1}{n^2} + \cdots = \frac{\pi^2}{6}.$$

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/