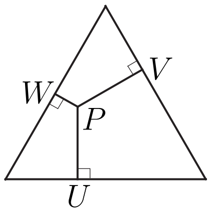
Problems, April 2006

Problem 1. When a certain Egyptian pyramid was built, the volume of the topmost 40 metres of the pyramid was equal to the volume of the bottom 1 metre. How high was this pyramid?

Problem 2. A rich merchant died, and left gold coins to his children as follows. The eldest got 100 coins, plus one-fifteenth of what remained of the coins after that. The next got 200 coins, plus one-fifteenth of what remained after that. And the next got 300 coins, and one-fifteenth of what remained, and so on. As it turned out, all the gold coins were distributed, and everyone got exactly the same amount. Is such an estate distribution possible? If so, how many children were there, and how much did each get? Generalize.

Problem 3. Let a, b, c, d, and e be integers, and let $P(x)$ be the polynomial $ax^4 + bx^3 + cx^2 + dx + e$. Show that if $P(x)$ is divisible by 5 for every integer x, then a, b, c, d, and e are all divisible by 5.

Problem 4. A point P is chosen at random inside an equilateral triangle. Perpendiculars PU, PV, and PW are drawn through P to the sides of the triangle. What is the probability that the three line segments PU, PV, and PW can be arranged to form a triangle? (By "at random" we mean here that if $\mathcal{R}$ is a region inside the triangle, the probability that P is in $\mathcal{R}$ is proportional to the area of $\mathcal{R}$.)



Problem 5. (a) Toss a fair coin six times. You win if there is a run of three *or more* heads or tails in a row. Find the probability that you win. (b) What about if you toss the coin eleven times? (This may be much harder.)

http://www.pims.math.ca/education/math_problems/
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