

Problems, April 2008

Problem 1. Let \mathcal{Q} be the convex quadrilateral whose vertices are the points where the curves with equations $x^4 + y^4 = 5$ and $xy = 1$ meet. Find as simple an *expression* as you can for the area enclosed by \mathcal{Q} .

Problem 2. Let S be the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. How many ordered pairs (A, B) are there such that (i) A and B are subsets of S , and (ii) $A \cap B = \emptyset$ (the intersection of A and B is the empty set, that is, A and B have no element in common). Note that the empty set is a subset of any set.

Problem 3. Let $P(x)$ be a non-constant polynomial with real coefficients. Show that there is an irrational number θ such that $P(\theta)$ is irrational.

Problem 4. Show that the sum of any number of consecutive perfect cubes is never prime.

Problem 5. Draw a line with negative slope through the point $(4, 2)$. Let X be the x -intercept of this line, let Y be the y -intercept, and let O be the origin. What is the smallest possible area of $\triangle OXY$? No calculus please.