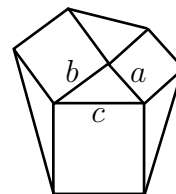


Problems, April 2009

Problem 1. The National Basketball Association (NBA) has 30 teams; each plays 82 games during the regular season. Call the teams $1, 2, \dots, n$, where $n = 30$. Let W_k be the number of regular season wins by team k , and let L_k be the number of regular season losses by team k . Show that

$$W_1^2 + W_2^2 + \dots + W_n^2 = L_1^2 + L_2^2 + \dots + L_n^2.$$

Problem 2. Outward facing squares are erected on the three sides of an arbitrary triangle. Then pairs of vertices of the squares are joined as in the figure below, forming a convex hexagon. Show that if the sides of the original triangle are a, b , and c , then the sum of the squares of the sides of the hexagon is equal to $4(a^2 + b^2 + c^2)$.



Problem 3. For which positive integers n is $4^n + n^4$ prime?

Problem 4. (a) Let $Q(x) = x^2 - x + 1$. Show that if m is an integer greater than 1 such that m divides a , then m divides none of $Q(a)$, $Q(Q(a))$, $Q(Q(Q(a)))$, and so on. (b) Use part (a) to prove that there are infinitely many primes.