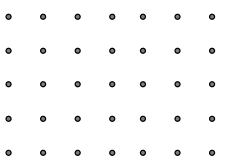
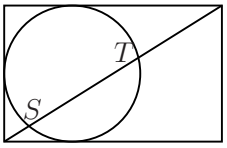
Problems, April 2010

Problem 1. In the rectangular grid below, each point is at distance 1 from its horizontal and vertical neighbours. We pick three non-collinear grid points A, B, and C. What are all the possible values of the area of $\triangle ABC$?



Problem 2. The picture is of a rectangle whose base is greater than its height. A circle is drawn in the rectangle, which is tangent to the top, left, and bottom edges. The diagonal from the bottom left corner to the top right corner meets the circle in points S and T. Express the distance ST in terms of the base and height.



Problem 3. Define $F(n)$ by

$$F(n) = \left(1 + \frac{2}{3}\right)\left(1 + \frac{4}{9}\right)\left(1 + \frac{16}{81}\right)\cdots\left(1 + \frac{2^{2^n}}{3^{2^n}}\right).$$

Find the smallest number a such that $F(n) \leq a$ for all n.

Problem 4. It is not difficult to verify that in the triangle with sides 4, 5, and 6, one of the angles is exactly twice another one. Find integers a, b, and c, which are sides of a triangle in which one angle is exactly twice another, such that the integers a, b, and c have no common divisor greater than 1, and all the integers are greater than 10.

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http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/