Problems, April 2011

Problem 1. If A, B, C, D, and E are any 5 points in the plane, no 3 of which lie on a line, let $m(A, B, C, D, E)$ be the measure (in degrees) of the smallest angle determined by 3 of these points. What is the largest possible value of $m(A, B, C, D, E)$?

Problem 2. For any real number u, let $\{u\}$ be the *fractional part* of u, that is, $\{u\} = u - \lfloor u \rfloor$, where $\lfloor u \rfloor$ is the greatest integer which is less than or equal to u. Find all real numbers x such that $\{(x-1)^3\} = \{(x+1)^3\}$.

Problem 3. Every point in the plane is assigned one of two colours, red or blue. Show that if some positive real number does not occur as a distance between two red points, then *every* positive real number occurs as a distance between two blue points.

Problem 4. The *diameter* of a well-behaved region in the plane is the largest possible distance between two points in the region. What is the diameter of the region enclosed by the figure made up of a 1×1 square surmounted by a semicircle of diameter 1? Preferably, calculus should not be used.