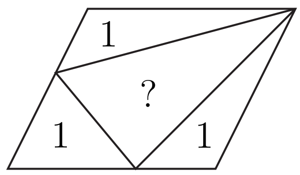
Problems, May 2006

Problem 1. In the figure below, a parallelogram has been divided into four triangles. If the areas of the three "outer" triangles are each 1 as shown, what is the area of the fourth triangle?



Problem 2. Find the last non-zero digit of 1000!.

Problem 3. (a) Find numbers A, B, and C such that

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

for all $x \neq 0$, -1, or -2.

(b) Simplify:

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{99 \cdot 100 \cdot 101}.$$

Problem 4. Find all pairs (x, y) of real numbers that satisfy the two equations

$$x + y = 1$$
$$x^5 + y^5 = 11.$$

Problem 5. Call a set $\mathcal{S}$ of positive integers *multiple-rich* if for any positive integer n, some multiple of n (perhaps n itself) is in $\mathcal{S}$. For example, the set of positive perfect squares is multiple-rich, and the set of primes is not.

Suppose that the positive integers are divided into two teams, say $\mathcal{A}$ and $\mathcal{B}$. Show that at least one of $\mathcal{A}$ or $\mathcal{B}$ is multiple-rich.

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/