Problems, May 2007

Problem 1. A circle meets the parabola $y = x^2$ at four points. The x-coordinates of three of the points are 2, 3, and 4. Find the x-coordinate of the fourth point.

Problem 2. In how many ways can one distribute 6 apples, 7 bananas, and 8 cantaloupes between 10 people? All fruit of the same type are identical. Fruit must remain whole, must all be given out, and possibly Alphonse gets everything.

Problem 3. For any positive integer n, let $f(n)$ be the largest odd divisor of n. Evaluate the sum

$$f(1001) + f(1002) + f(1003) + \cdots + f(1999) + f(2000).$$

Problem 4. (a) Points P and Q are chosen on the curve $x^2 + 4y^2 = 1$ in such a way that the distance PQ is as large as possible. Find that distance. (b) Solve the same problem for the curve $x^4 + 16y^4 = 1$.

Problem 5. Let α, β, and γ be the solutions of $x^3 - 3x + 1 = 0$. Find

$$(2\alpha - 1)(2\beta - 1)(2\gamma - 1).$$

http://www.pims.math.ca/education/math_problems/
http://www.math.ubc.ca/~adler/problems/