

Problems, May 2008

Problem 1. Alphonse and Beti work together to separate suckers from their money. Beti leaves the room. The mark (Mark) is asked to toss a loonie and a toonie. Alphonse hands one of the tossed coins back to Mark, and bets Mark \$20 that Beti can guess whether that coin landed head or tail. Beti comes back in, and guesses correctly. How? If the game is played many times, how can Alphonse and Beti make it harder for Mark to know *how* he is being fleeced?

Problem 2. For any real number u , let $\lfloor u \rfloor$ be the greatest integer which is less than or equal to u . For example, $\lfloor 17/5 \rfloor = 3$. How many integers n in the interval from 0 to 999 (inclusive) can be expressed in the form

$$n = \lfloor x \rfloor + \lfloor 2x \rfloor + \lfloor 3x \rfloor + \lfloor 4x \rfloor,$$

where x is a real number?

Problem 3. A triangle of area 1 has sides a , b , and c , where $a \leq b \leq c$. Find the least possible value of b .

Problem 4. Find all ordered pairs (a, b) of integers such that $a^2 - 4b$ and $b^2 - 4a$ are both perfect squares. In this kind of problem, one needs to *prove* that all solutions have been found.

Problem 5. We are given a set \mathcal{S} of 200 points in the plane, no three collinear. Of these points, 100 are white, and 100 are black. Show that there exists a collection of 100 line segments such that (i) the endpoints of any line segment lie in \mathcal{S} and are of different colours and (ii) no two of the line segments meet.