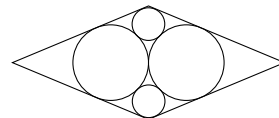


Problems, May 2010

Problem 1. The figure below is a rhombus whose diagonals have lengths 72 and 30. Two congruent “large” circles are drawn, and then two congruent small circles, with tangencies as shown. What is the radius of a small circle?



Problem 2. The sequence $a_0, a_1, a_2,$ and so on is defined by $a_0 = 2$ and $a_{n+1} = (2a_n + 1)/(a_n + 2)$ for $n \geq 0$. Find an explicit formula for a_n , and prove that the formula is correct.

Problem 3. (a) Find a simple expression for

$$1 \cdot \binom{n}{1} + 2 \cdot \binom{n}{2} + 3 \cdot \binom{n}{3} + \cdots + n \cdot \binom{n}{n}.$$

(b) Find a simple expression for

$$(2)(1) \binom{n}{2} + (3)(2) \binom{n}{3} + (4)(3) \binom{n}{4} + \cdots + (n)(n-1) \binom{n}{n}.$$

Problem 4. Is $\lfloor (1 + \sqrt{2})^{999} \rfloor$ odd or is it even? (As usual, $\lfloor x \rfloor$ is the greatest integer which is $\leq x$.)