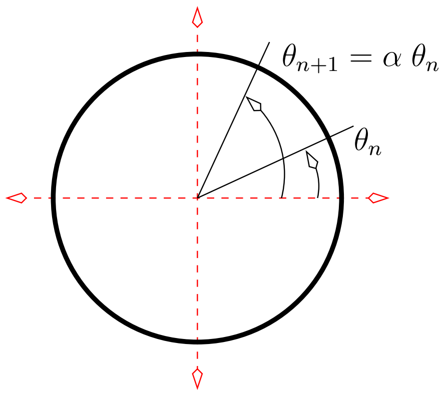
Assignment 1

- **Due: Wednesday April 2 @ 5pm.**
- **Remember to fill in and attach a plagiarism coversheet.**
- **Put in the assignment box in the Maths department.**

1. Consider the dynamical system on the circle S^1 defined by:

$$\theta_{n+1} = \alpha\, \theta_n.$$



Describe the dynamics of the system for values of $\alpha \geq 0$. Please include discussion of:

- fixed points (and their nature — *i.e.* attracting, repelling or neutral),
- periodic points (and their nature),
- "sensitive dependence on initial conditions",

and anything else you feel is relevant.

2. Consider the dynamical system on the circle S^1 defined by:

$$\theta_{n+1} = 2\, \theta_n.$$

- Prove that the set of all periodic points of this system is dense in the circle S^1.
- Also prove that the set of points that are *not* eventually periodic is also dense in S^1.

3. An experimental investigation of rates of convergence: Using a computer investigate (numerically) how quickly an orbit is attracted to a fixed point.

   Procedure: Each of the functions listed below has a fixed point and the orbit of $x_0 = 0.2$ is attracted to it. For each function listed below use a computer (I have provided an applet on the subject homepage) to compute the orbit of $x_0 = 0.2$ until it reaches the fixed point — or within 10^{-5} of it.

   For each function you should make note of:

   (a) the location of the fixed point, p,
   (b) the derivative at the fixed point, $f'(p)$,
   (c) is the fixed point attracting or neutral,
   (d) the number of iterations it took for the orbit of 0.2 to reach (within 10^{-5}) p.

   The functions in question are:

   (a) $f(x) = x^2 + 0.25$
   (b) $f(x) = x^2$
   (c) $f(x) = x^2 - 0.26$
   (d) $f(x) = x^2 - 0.75$
   (e) $f(x) = 0.4x(1 - x)$
   (f) $f(x) = x(1 - x)$
   (g) $f(x) = 1.6x(1 - x)$
   (h) $f(x) = 2x(1 - x)$
   (i) $f(x) = 2.4x(1 - x)$
   (j) $f(x) = 3x(1 - x)$
   (k) $f(x) = 0.4 \sin x$
   (l) $f(x) = \sin x$

   Results: When you have collected the data, compare each of the functions. Describe what you observe — in particular the relationship between the speed of convergence and $f'(p)$.