Assignment 2

- **Due: Tuesday April 29 @ 5pm.**
- **You only need to attach a plagiarism coversheet if you didn't submit the first assignment.**
- **Put in the assignment box in the Maths department.**
- **Please be neat! Many people handed in appallingly messy work for assignment 1. — I will deduct up to 50% for messy work.**

1. Using the java applets on the course homepage, calculate the locations of the first 6 *superstable* orbits of:

   - The logistic map $F_\mu(x) = \mu x(1-x)$, and
   - The sine map $S_\mu(x) = \mu \sin(\pi x)$

   A point x_0 is a superstable stable n-cycle if it has $(F^n)'(x_0) = 0$. Since both these maps have a unique maximum at $x = 1/2$, the superstable n-cycles occur at μ values where the n-cycle contains the point $x = 1/2$. See next page for an example.

   Remember — these maps undergo period doubling bifurcations, so the first 6 superstable orbits will have periods $1, 2, 2^2, \ldots 2^5$. It is important that your estimates are *very* accurate — as many decimal places as you can get! Take your time and be careful. You will need to set "Discard first" and "Show next" to around 100,000 in order to get good results.

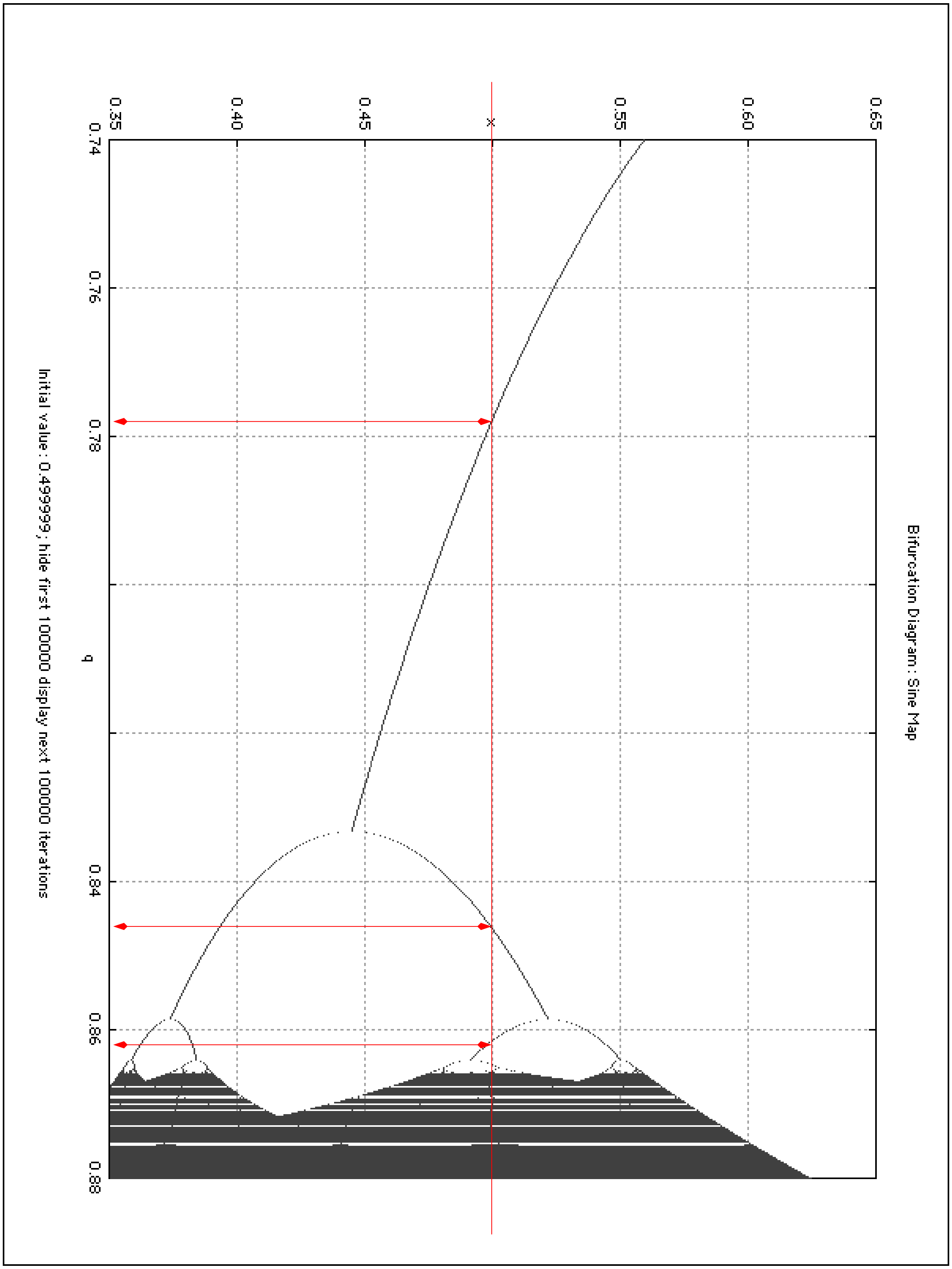
   Call these μ values for the logistic and sine maps $l_0, l_1, \ldots l_5$ and $s_0, s_1, \ldots s_5$ (respectively). To check you have the right idea — $l_0 = 2$ and $s_0 = 1/2$. These numbers are expected to have the following behaviour:

$$\begin{aligned} l_n &\approx l_\infty - A\delta_l^n \\ s_n &\approx s_\infty - B\delta_s^n \end{aligned}$$

   Tabulate your data and estimate δ_l and δ_s. What do your results suggest?

   Hint: If $l_n \approx l_\infty - A\delta_l^n$ then

$$\begin{aligned} l_n - l_{n-1} &\approx A\delta^{n-1}(\delta - 1) \\ \frac{l_n - l_{n-1}}{l_{n-1} - l_{n-2}} &\approx \delta \end{aligned}$$


Bifurcation Diagram: Sine Map
Initial value: 0.499999; hide first 100000 display next 100000 iterations
q
x
0.74
0.76
0.78
0.84
0.86
0.88
0.35
0.40
0.45
0.55
0.60
0.65

2. Let Σ_N denote the space of sequences whose entries are the integers $0, 1, \ldots, N-1$.

   (a) For all $s, t \in \Sigma_N$ define

   $$d[s,t] = \sum_{k=0}^{\infty} \frac{|s_k - t_k|}{N^k}$$

   Prove that this function is a metric on Σ_N, and find the maximum distance between any two points in Σ_N.

   (b) Let σ_N be the shift map on Σ_N (defined in the usual way). How many fixed points does σ_N have? How many points are fixed by σ_N^k?