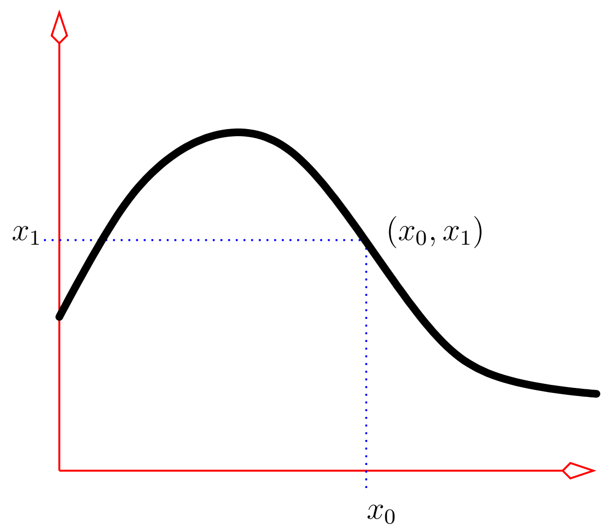
2 Graphical Analysis — summary

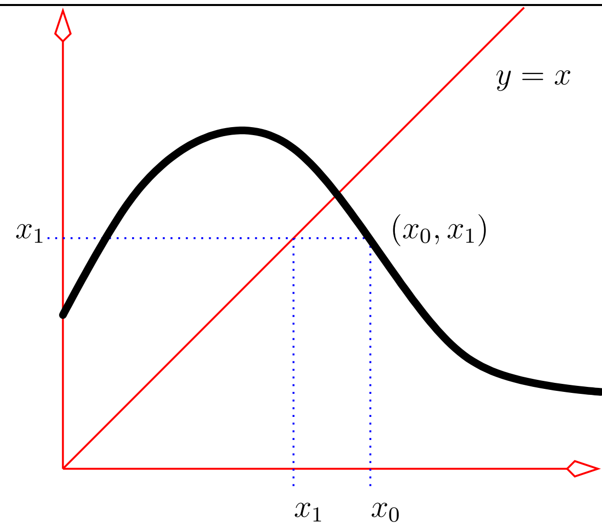
2.1 Cobweb diagrams

Cobweb diagrams allow us to iterate a function by entirely graphical means and without having to resort to analytic or numerical methods.

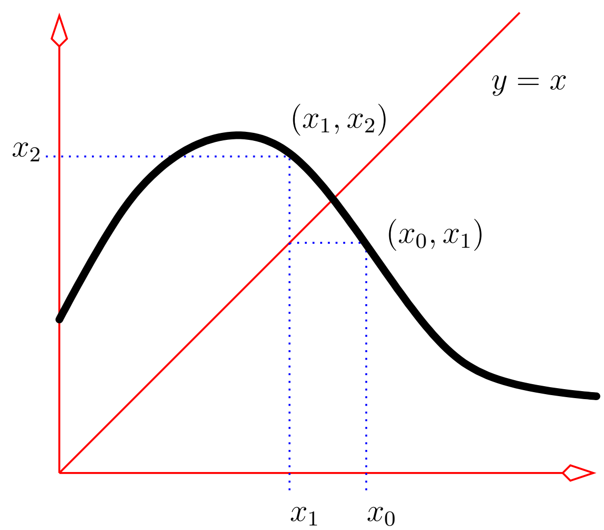
Consider the function $f(x)$ plotted below. Starting from a point x_0, we can find the next iterate of the function, $x_1 = f(x_0)$, simply by drawing a vertical line to the plot of the function. x_1 can be then marked on the vertical axis by drawing a horizontal line from the point of intersection.



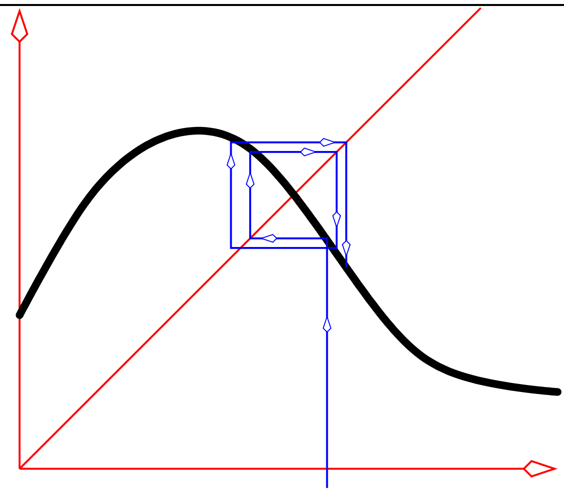
In order to find $x_2 = f(x_1)$, we need to move the point x_1 marked on the vertical axis to the same point on the horizontal axis. We do this by finding the intersection of the horizontal line with the line $y = x$. Since the horizontal line has equation $y = x_1$, this intersection will occur at the point (x_1, x_1). Drawing a vertical line down to the horizontal axis will then mark the point x_1.



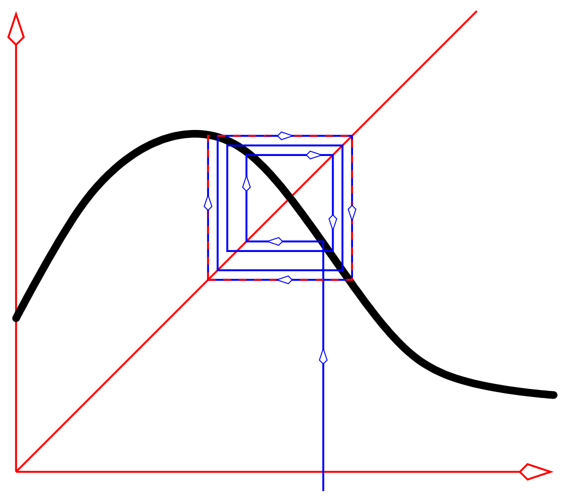
Given that we now have x_1 on the horizontal axis we can find the point $x_2 = f(x_1)$ by drawing a vertical line up to the plot of the function.



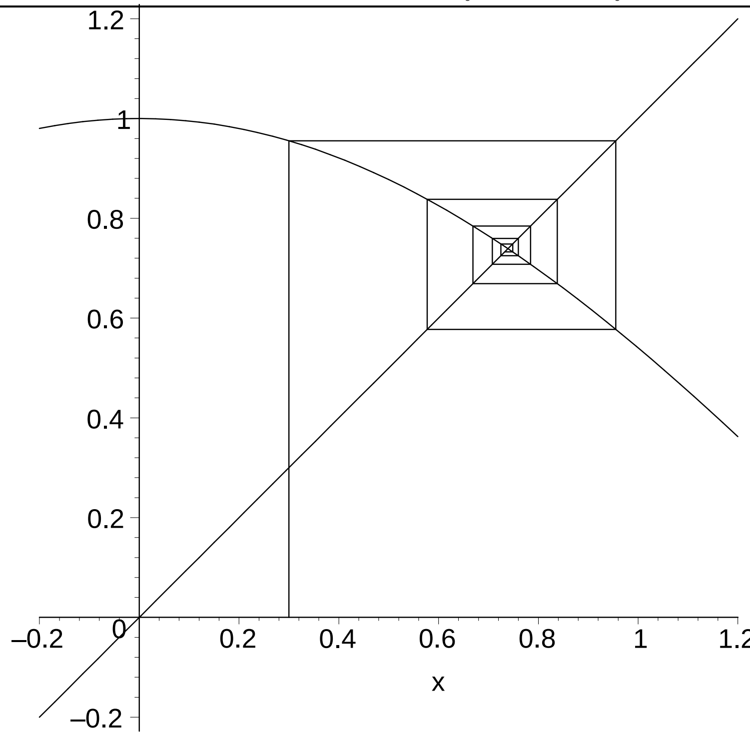
This proceedure can then be iterated to generate a "*cobweb diagram*" which shows the positions of future iterates of the function.



In this example we see that the iterates settle into a 2-cycle (which is marked in blue and red).

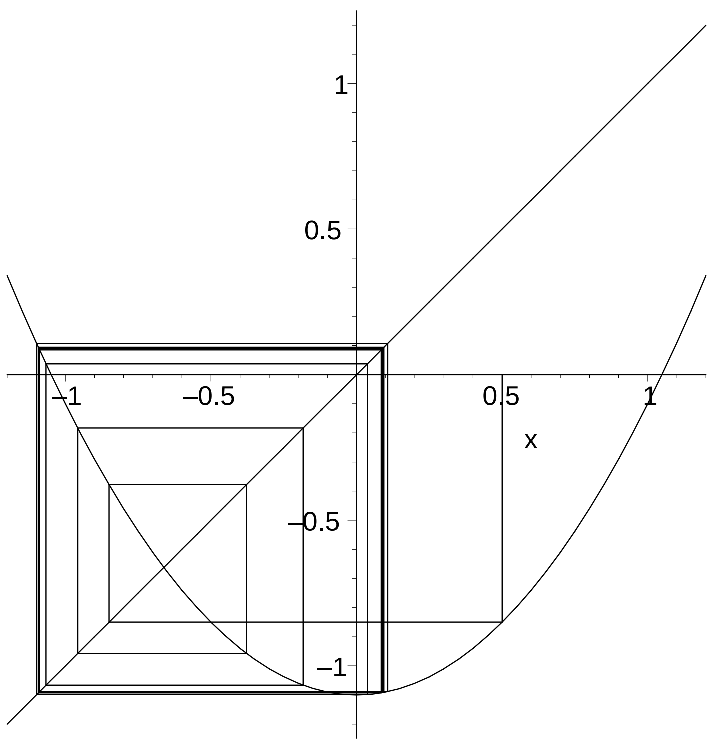


Below is a cobweb diagram of the iterates of $x_0 = 0.3$ under $\cos(x)$:



We see that the iterates converge to the fixed point at $x \approx 0.739\ldots$

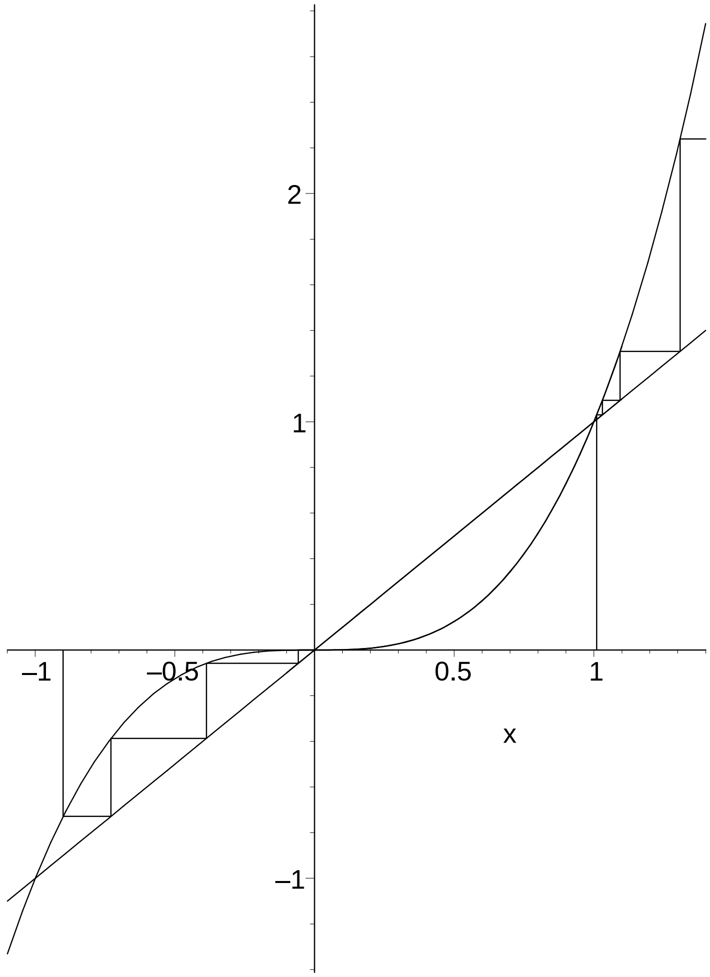
Below is a cobweb diagram of the iterates of $x_0 = 0.5$ under $x^2 - 1.1$:



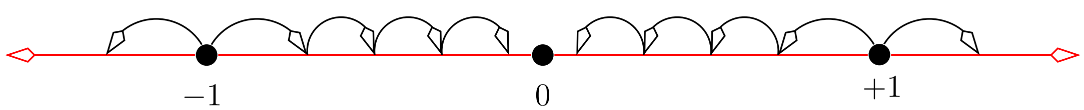
We see that the iterates converge to the two cycle at $\{p_+, p_-\} \approx \{0.0916, -1.0916\}$.

2.2 Phase portraits

In the picture below we show the iterates of $f(x) = x^3$. This function has three fixed points, $x = -1, 0, 1$. If $|x_0| > 1$ then iterates diverge to infinity, while if $|x_0| < 1$ then the iterates converge to the fixed point at $x = 0$.



We can summarise the information about these orbits on the real line by the following diagram:



This is a "*phase portrait*". It shows how points on the real line move to new points on the real line under application of $f(x)$. While this diagram gives no more information than the cobweb diagram, we are able to use phase portraits for higher dimensional systems where we are unable to apply cobweb diagrams.