1 Problem Set 1 — Orbits

1. Let $F(x) = x^2$. Compute the first five points of the orbit of $1/2$.

2. Let $F(x) = x^2 - 1$. Compute $F^2(x)$ and $F^3(x)$.

3. Find all the real fixed points of the following functions

   (a) $F(x) = 3x + 2$

   (b) $F(x) = x^2 - 2$

   (c) $F(x) = x^3 - 3x$

   (d) $F(x) = |x|$

   (e) $F(x) = x \sin x$

4. Find the fixed points and two-cycles of the function $F(x) = 1 - x^2$.

The following questions correspond to the *doubling map*,

$$D : [0, 1) \mapsto [0, 1)$$

defined by

$$D(x) = \begin{cases} 2x & 0 \leq x < 1/2 \\ 2x - 1 & 1/2 \leq x < 1 \end{cases}$$

which is equivalent to

$$D(x) = 2x \mod 1.$$

5. Discuss the orbits of the following points under $D(x)$:

   (a) $x_0 = 0.3$

   (b) $x_0 = 0.7$

   (c) $x_0 = 1/8$

   (d) $x_0 = 1/7$

   (e) $x_0 = 3/11$

6. Explain why a computer might have difficulty computing the orbit of $1/7$ if you give a decimal expansion (*Hint* — computers store numbers in binary).

7. Write down an explicit formula for $D^2(x)$. Draw a graph of $D(x)$, $D^2(x)$ and $D^3(x)$.

8. Find all the fixed points of $D(x)$, $D^2(x)$ and $D^3(x)$. How many fixed points does $D^n(x)$ have?

The following questions correspond to the *tent map*,

$$T : [0, 1] \mapsto [0, 1]$$

defined by

$$T(x) = \begin{cases} 2x & 0 \leq x \leq 1/2 \\ 2 - 2x & 1/2 < x \leq 1 \end{cases}$$

9. Sketch $T(x)$ — the name of this function should become obvious.
10. Find a formula for $T^2(x)$ and sketch this function.
11. Find all fixed points of $T(x)$ and $T^2(x)$.
12. What does the graph of $T^n(x)$ look like and how many fixed points does it have?