Stochastic Processes

Sample midterm 2

Note: These are sample questions, and is not an indication of the midterm length.

Problem 1. Let $\{N(t)\}_{t\geq 0}$ be a rate λ Poisson process, and $s < t$ postitive numbers.

(a) Find $P(N(t) > N(s))$
(b) Find $P(N(s) = 0, N(t) = 3)$.
(c) Find $E[N(t)|N(s) = 4]$.
(d) Find $E[N(s)|N(t) = 4]$.
(e) Find the joint probability mass function of $N(t), N(t)$.
(f) What is $P(N(1) = 1, N(2) = 2, N(3) = 3, N(4) = 4)$?
(g) Let S_i be the time of the ith event in N. What is $E(S_n)$?
(h) For a given t, what is $E(S_1|N(t) = 2)$
(i) For a given t, what is $E(S_4|N(t) = 2)$
(j) Suppose that each event is type 1 with probability p, and type 2 with probability $1-p$. Let $N_i(t)$ be the number of type i events by time t. What is $E[N(t)|N_1(t+s) = k]$ for some integer k?

Problem 2. Starting from time 0, the #99 bus arrives according to a rate λ Poisson process. Passengers arrive at the station according to an independent rate μ Poisson process. When a bus arrives, all waiting passengers instantly enter the bus.

(a) Find the p.m.f. for the number of passengers entering a bus.
(b) Find the p.m.f. for the number of passengers entering the a bus given that the time between it and the previous bus is t.
(c) Suppose that the buses have been going for ever. You arrive at the bus stop at 9:00. What is the distribution of the time since the last bus had gone?
(d) You arrive at the bus stop at 9:00. What is the expected number of passengers waiting at the station?

Problem 3. A small barbershop has room for at most two customers (including the one the barber is working on). Potential customers arrive at a Poisson rate of 3 per hour, and the successive service times are independent exponential random variables with mean 4 per hour. If a customer arrives and there is no room in the shop, they leave.

Set this up as a birth and death process: Find the rates for jumping out of each state and the jump probabilities.