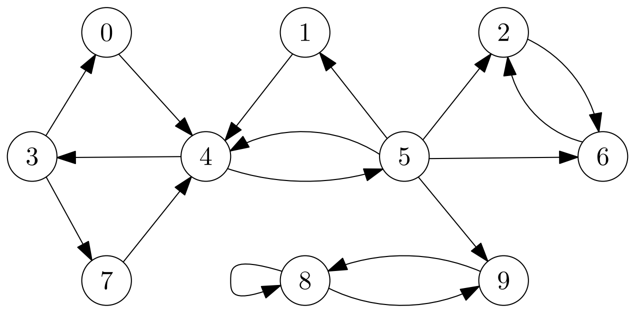
Stochastic Processes
Assignment 2, due 2022-01-28

Problem 1. Consider the chains with state space the squares of a chess
(a) A chess king moves on it uniformly to an allowed position. Is this irreducible? periodic? recurrent?
(b) Same questions if the king is replaced by a bishop.
(c) Same questions if the king is replaced by a knight.

Problem 2. Consider the Markov chain with transition diagram below (all arrows have unspecified positive probability. What are the communicating classes? For each class determine if it is recurrent or transient and the period.



Problem 3. Let P be the transition matrix for an irreducible Markov chain. The lazy chain has the transition matrix $\hat{P} := (1-a)I + aP$ for some constant $a \in (0,1)$. Show that the following hold.
(a) $\hat{P}$ is a valid transition matrix, i.e., it is a stochastic matrix.
(b) If a Markov chain is irreducible and has some state i with $P_{i,i} > 0$, show that the chain is aperiodic. (In particular this applies for $\hat{P}$.)
(c) $\hat{P}$ has the same stationary distributions as P.

Problem 4. Consider a Markov chain with n states arranged in a circle. At each step the chain jumps one step clockwise with probability $2/3$ and one step anticlockwise with probability $1/3$.
(a) Show that this is periodic if n is even and aperiodic if n is odd.
(b) What is the stationary measure for this chain?

Problem 5. Recall the chain from last week: There are 6 coins on a table, each showing heads (H) or tails (T). In each step we
- Select uniformly one of the coins.
- If it is heads, toss it and replace on the table (with random side).
- If it is tails, toss it. If it comes up heads leave it at that. If if comes up tails, toss it a second time, and leave the result as it is.

Let X_n be the number of heads showing after n such steps.
Show that the stationary distribution is $\mathrm{Bin}(6, q)$ for some q.

Extra practice problems Do not hand these in. (Feel free to ask for hints is stuck.)
Ross, chapter 4: problems 14,16,18,21,38.