Stochastic Processes

Assignment 5, due 2022-03-04

Note: Start each problem on a **new page**.

Problem 1. Suppose X is an exponential random variavle with unknown rate λ.
(a) Find a for which $P(a \leq X \leq 2a)$ is maxized.
(b) If $P(X \leq 5) = \frac{2}{3}$, find $P(X \geq 10)$, and find λ.

Problem 2. Suppose X, Y are independent exponential variables with rates λ, μ respectively.
(a) Prove that $P(X \leq Y) = \frac{\mu}{\lambda+\mu}$.
(b) Calculate the conditional distribution of X given that $X \leq Y$.

Problem 3. Recall the bank example from class. Suppose service at teller A takes $\text{Exp}(\lambda_a)$ time and service at teller B takes $\text{Exp}(\lambda_b)$ for some λ_a, λ_b. Naine arrives at the bank when the two tellers are serving customers, but no others are in line. Show that the probability that Naine leaves the bank after both previous customers is $2\frac{\lambda_a\lambda_b}{(\lambda_a+\lambda_b)^2}$.

Hint: use the previous problem.

Problem 4. Oaine is having office hours and Paine and Qaine will show up. The arrival times are independent exponentials with rates λ_P and λ_Q. After arriving they stay for exponential times with rates μ_P and μ_Q respectively (all independent).
(a) What is the probability that Paine comes and leaves before Qaine arrives?
(b) What is the expected time for the last student to leave?

Problem 5. For a Poisson process $N(t)$ with rate λ, find the following:
(a) $P(N(t) = n | N(s) = m)$ for $m \leq n$ and $s \leq t$.
(b) $P(N(s) = m | N(t) = n)$ for $m \leq n$ and $s \leq t$.

Extra practice problems Do not hand these in. (Feel free to ask for hints is stuck.)

Ross, chapter 5: problems 3,7,10,18,23,26,39.

Read ahead We will finish Chapter 5 next week (Sections 5.3 and a part of 5.4).