Stochastic Processes

Assignment 6, due 2022-03-11

Note: Start each problem on a **new page**.

Problem 1. In a pizzeria the orders arrive according to a Poisson process of rate λ. Given that 3 orders arrived in an hour, determine the conditional probability density functions of the arrival of the first times S_1, S_2, S_3. (Note: this asks for 3 probability density functions, not the joint p.d.f.)

Problem 2. Let $N(t)$ be a Poisson process of rate λ. Determine the conditional distribution of $N(3)$ given that $N(5) = 7$.

Problem 3. Raine is a birdwatcher and counts the birds in her garden, which arrive according to a Poisson process of rate λ. If an hour passes with no birds, she gets bored and goes to make tea.

(a) What is the probability that she doesn't see any birds?
(b) How much time on average does she spend birdwatching? Hint: Condition on the time of the first observed bird.
(c) What is the distribution of the number of birds seen?

Problem 4. Players arrive at a casino at rate λ, and begin betting, and losing money at rate \$1/minute. Let X be the total amount they lose by time t.

(a) What is $E(X|N(t))$?
(b) Explain why $\mathrm{Var}(X|N(t)) = N(t)t^2/12$.
(c) Find $\mathrm{Var}(X)$.

Problem 5. A system suffers faults by a Poisson process with rate λ. Each fault will, independently, cause the system to fail after time X with c.d.f. F. When the system fails, the responsible fault is fixed, but earlier other faults may remain and cause later failure.

(a) What is the distribution of the number of failures up to t?
(b) What is the distribution of the number of faults that remain in the system at time t and are not yet fixed?
(c) Are the random variables in parts (a) and (b) dependent or independent? (justify!)

Extra practice problems Do not hand these in. (Feel free to ask for hints is stuck.)
Ross, chapter 5: problems 50, 53, 56, 64,68, 71.

Read ahead We will start Chapter 6 next week.