Stochastic Processes

Assignment 7, due 2022-03-25

Note: Start each problem on a **new page**.

Problem 1. Potential customers arrive at a queue at times of a Poisson process with rate λ. If a customer sees n others in the queue, the join with probability α_n, and give up and go home with probability $1-\alpha_n$. The service rate is always μ. Set this up as a birth and death process and determine the birth and death rates.

Problem 2. Consider two machines, both of which have an exponential lifetime with parameter λ. There is a single repairman can service machines at an exponential rate μ. Let X_t be the number of machines that are operational at time t. Write the Kolmogorov backward equations (you need not solve them).

Problem 3. Consider a Yule process with parameters $\lambda_n = n\lambda$ and $\mu_n = 0$. Start the process with $X_0 = 1$ individual. Let T_n be the time at which the population reaches size n (so $T_1 = 0$).
(a) Explain why $T_{n+1} - T_n$ are independent exponential variables, and give their parameters.
(b) Find $E(T_n)$. (You do not need to simplify sums.)

Problem 4. A continuous time Markov chain on states $\{a, b, c\}$ has jump rates

$$q_{ab} = 1 \qquad q_{ba} = 2 \qquad q_{ca} = 2$$
$$q_{ac} = 1 \qquad q_{bc} = 1 \qquad q_{cb} = 2.$$

Suppose $X_0 = a$. We wish to find the expected time to reach state b, denoted T_b.
(a) What is the expected time for the first jump out of a?
(b) Once the chain leaves a, what is the distribution of the next state?
(c) Let M_i be the expected time to reach b if we start at state i. (So $M_b = 0$). Use parts (a),(b) to write an equation for M_a in terms of M_b and M_c.
(d) Write a similar equation for M_c.
(e) Solve the equations to find M.

Problem 5 (Ross, 6.2)**.** A one-celled organism can be in one of two states: A or B. An individual in state A will change to state B at an exponential rate α. An individual in state B divides into two new individuals of type A at an exponential rate β. Define an appropriate continuous-time Markov chain for a population of such organisms and determine the appropriate parameters for this model (the transition rates and jump probabilities.) (Hint: the state (N_A, N_B) is the number of individuals of each type.)

Extra practice problems Do not hand these in. (Feel free to ask for hints if stuck.)
Ross, chapter 6: problems 1,3,4,9,12.

Read ahead We will continue Chapter 6 next week, towards limiting probabilities.

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