

MATH 318 midterm

February 2023

- The test consists of 10 pages and 5 questions worth a total of 54 marks.
- This is a closed-book examination. Calculators are allowed. **None of the following are allowed:** documents, cheat sheets or other electronic device of any kind (cell phones, etc.)
- Use each page **only** for the question indicated.
- **If you need more paper:** ask an invigilator. You must write the 4-digit test number and the question number on both sides of any extra papers to be marked.
- Justify all claims.
- Questions are **not** in order of difficulty.
- Time allowed: 50 minutes.
- Good luck, enjoy the break.

Student number								
Section								
Name							
Signature								

Scratch work – will not be marked.

1. 12 marks A random variable X is uniform in $\{1, 2, 3, 4, 10\}$.

(a) What is the expectation $E[X]$?

(b) What is the variance $\text{Var } X$?

(c) What is the characteristic function of X ?

2. 12 marks We toss five dice (all 6-sided).
- (a) What is the probability of getting exactly three 1s?

- (b) What is the probability of getting five different numbers?

Next, we take four cards from a standard deck (13 cards in each of 4 suits).

(c) What is the probability of getting exactly three hearts?

(d) What is the probability of getting one card from each suit?

3. 8 marks Let $X \sim N(0, 1)$ be a standard normal random variable, and let $Y = X^2$.
- (a) Write the cdf of Y in terms of Φ , where $\Phi(t) = P(X \leq t)$ is the normal cdf.

- (b) Find a formula for the pdf of Y .

4. 8 marks The king has three urns, each with some balls. The first (A) contains 1 white and 1 black ball. The second (B) contains 2 white and 3 black balls. The third (C) contains 4 white and 1 black balls. Each prisoner picks a random urn, and takes out a ball at random without looking. If they pick a black ball, they will be hanged. If they pick a white ball, they will be drowned.

(a) What is the probability that a prisoner is hanged?

(b) A prisoner ends up with a black ball. What is the conditional probability that they picked urn A?

5. 14 marks Random variables X, Y have joint pdf $f(x, y) = x + 2y^3$ on $[0, 1] \times [0, 1]$ and 0 outside that square.

(a) What is the marginal distribution of X ?

(b) What is the marginal distribution of Y ?

(c) Are they independent? (justify!)

(d) What is $\text{Cov}(X, Y)$? (Write the needed integrals; compute them if you have time.)

Table 1: Common Distributions

Distribution	Mean	Variance	Characteristic function
Bin(n, p)	np	$np(1 - p)$	$(1 - p + pe^{it})^n$
Geom(p)	$1/p$	$\frac{1 - p}{p^2}$	$\frac{pe^{it}}{1 - (1 - p)e^{it}}$
Poi(λ)	λ	λ	$e^{\lambda(e^{it} - 1)}$
Unif[a, b]	$\frac{a + b}{2}$	$\frac{(b - a)^2}{12}$	$\frac{e^{ita} - e^{itb}}{it(b - a)}$
Exp(λ)	$1/\lambda$	$1/\lambda^2$	$\frac{\lambda}{\lambda - it}$
N(μ, σ^2)	μ	σ^2	$e^{i\mu t - \sigma^2 t^2/2}$

The normal CDF:

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990