2023-01-0

Combinations: How many ways to pick 2 out of $\{a, b, c, d\}$?

Ans: 6 $\{ab\}$ $\{ac\}$ $\{ad\}$ $\{bc\}$ $\{bd\}$ $\{cd\}$

$\frac{4 \cdot 3}{2!}$ ← ordered pairs
← forget the order.

To pick k out of n:

$$\frac{n \cdot (n-1)(n-2) \cdots (n-k+1)}{k!} = \frac{n!/(n-k)!}{k!} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

(n choose k) : binomial coefficient.

Counting

Permutations (orders)

Q: How many permutations of $\{a, b, c\}$?

Ans: $3! = 3 \cdot 2 \cdot 1 = 6$

abc bac cab
acb bca cba

For 4 letters: $4 \cdot 3! = 4! = 24$

For n letters: $n!$

Arrangements: Orders with repetition.

ALLELE: 

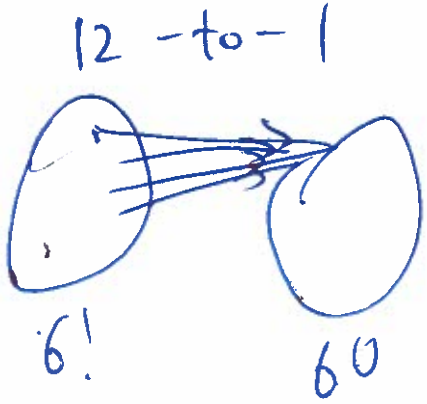
How many "words" can we get from these 6 letters?

$A L_1 L_2 E_1 L_3 E_2$: $6!$ permutations.

no diff. when permute the 2 E's or 3 L's

each word corresponds to $2! \cdot 3!$ permutations

so # of words is $\frac{6!}{2! \cdot 3!} = \frac{6!}{2 \cdot 6} = 60$



Note: $\binom{n}{k} = \binom{n}{n-k}$

Q: How many bridge hands are there?

(deal 52 cards to 4 players)

$$\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13} = \frac{52!}{13!\,39!} \cdot \frac{39!}{13!\,26!} \cdot \frac{26!}{13!\,13!} \cdot \frac{13!}{13!\,0!} = \frac{52!}{13!\,13!\,13!\,13!}$$

note: $0! = 1$

$\approx 5 \cdot 10^{28}$

More generally: to distribute n items to k boxes with $n_1, n_2, \ldots n_k$ in the boxes:

$$\frac{n!}{n_1!\ n_2!\ \cdots\ n_k!} = \binom{n}{n_1, n_2, \ldots, n_k}$$

multinomial coefficient

This is also the number of words if letters appear $n_1 \dots n_k$ times.

$$\binom{6}{1,2,3} = 60$$