Probability

Terminology
- Sample Space S or Ω
- Event E is a subset $E \subset S$

e.g. S = 5-card poker hands. $|S| = \binom{52}{5}$

E = one pair hands

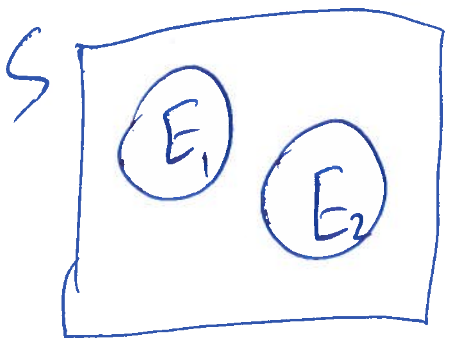
Repeat an experiment n times,

Let $N_n(E)$ be the number of times result in E

Observe that $\frac{N_n(E)}{n}$ conv. as $n \to \infty$ to $P(E)$

For any event $E \subset S$ we have $P(E)$ is its probab.

we always have $P(E) \geq 0$
$P(E) \leq 1$



$$\left.\begin{array}{l}\text{If } E_1 \cap E_2 = \phi \text{ then} \\ P(E_1 \cup E_2) = P(E_1) + P(E_2)\end{array}\right\} \text{Additivity for disjoint events}$$

$P(S) = 1$

$P(\phi) = 0$

Definition: A probability (measure) is a funct. P that assigns to each event E a number $P(E)$

s.t. ① $0 \le P(E) \le 1$ for all E.

② $P(S) = 1$

③ If $E_1, E_2, E_3 \ldots$ are disjoint (in pairs)

then $P(E_1 \cup E_2 \cup \ldots) = \sum_i P(E_i)$

note: can have finite or countable seq. of events for ③

For finite S, only care about finite sequences.

Probability space: $(S, \mathcal{F}, P)$

all possible events.

Often ⊛ S finite and $\mathcal{F} = \mathcal{P}(S)$ = all subsets.

power set

⊛ Often all outcomes are equally likely.
(Not always!)

In this case, $P(E) = \dfrac{|E|}{|S|}$

e.g. Find P(four of a kind)

S = 5-card hands, $|S| = \binom{52}{5}$

E = 4-of-a kind $|E| = 13 \cdot 48$

$$P(E) = \frac{13 \cdot 48}{\binom{52}{5}} \approx 0.00024$$

P(full house):

other rank

$$|E_2| = \underset{\text{3-of what}}{13} \cdot \underbrace{\binom{4}{3}}_{\text{which 3}} \cdot 12 \cdot \binom{4}{2}$$ which ones.

$$P(\text{full house}) = \frac{3744}{\binom{52}{5}} \approx 0.0014$$

Birthday problem:

n people are present. Do 2 have same birthday?

$(B_1, \ldots, B_n)$ are birthdays of n people.

$(B_1, \ldots, B_n) \in S$ = seq. of n dates

$$|S| = 365^n$$

$$P(\text{all different}) = \frac{365 \cdot 364 \cdot 363 \cdot 362 \cdots (366-n)}{365^n}$$

$$< \frac{1}{2} \quad \text{if} \quad n > 23$$