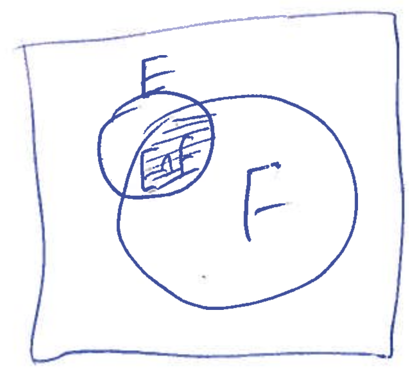
Conditional probability + Indepence.

Assume F has $P(F) > 0$.

The cond. prob $P(E|F)$ is the prob. of E given that F occurs

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$



Easy to check that $P(\cdot|F)$ satisfies the def. of probability.

$E \longrightarrow P(E|F)$

1) For any E $0 \le P(E|F) \le 1$

2) $P(S|F) = 1$

3) For disjoint $E_1, E_2, \dots$

$$\sum P(E_i|F) = P\left(\bigcup_i E_i \mid F\right)$$

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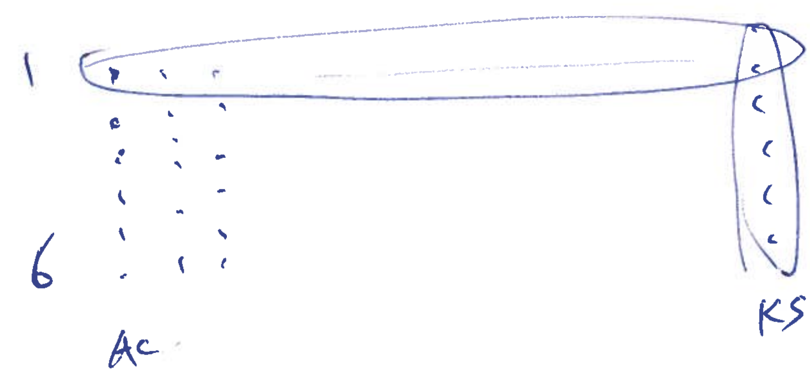
dice + card: $S = \{1 \dots 6\} \times \{\text{cards}\}$



__note__: $P(E \cap F) = P(F) \cdot P(E|F)$

__def__: events E, F are indep if $P(E \cap F) = P(E)\,P(F)$

$\iff P(E|F) = P(E)$ if $P(F) \neq 0$

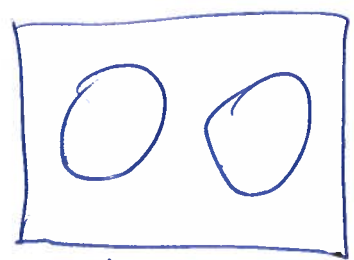
$\iff P(F|E) = P(F)$ if $P(E) \neq 0$

$$\underset{\frac{4}{52}}{P(\text{Ace})} \cdot \underset{\frac{1}{2}}{P(\text{red card})} = \underset{\frac{2}{52}}{P(\text{red ace})}$$

so E = red card, F = Ace are indep.

__note__: indep. and disjoint are different.

__disjoint__ $E \cap F = \emptyset$



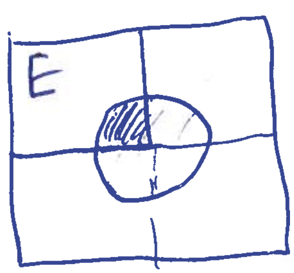
disjoint.

e.g. $P(A) = \frac{1}{100}$ $P(B) = \frac{1}{20}$ A, B indep.

what is $P(A \cup B)$

<u>Ans</u>: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ inc.-exc.

$= \frac{1}{100} + \frac{1}{20} - \frac{1}{100} \cdot \frac{1}{20}$ independent.

S 

E = quarter square

F = circle.

pick a point in S uniformly.

$P(E) = 1/4$ $P(E|F) = 1/4$ so E, F are indep.

def: events $E_1, E_2, E_3 \ldots$ are indep. if for any sub sequence $E_{i_1}, E_{i_2} \ldots E_{i_k}$ we have

$$P(E_{i_1} \cap E_{i_2} \ldots \cap E_{i_k}) = P(E_{i_1}) \cdot P(E_{i_2}) \cdots P(E_{i_k})$$

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e.g. toss two coins $S = \{HH, HT, TH, TT\}$

A = first is H $\quad P(A) = 1/2$

B = second is H $\quad P(B) = 1/2$

C = both are equal. $\quad P(C) = 1/2$

Any two are indep. but $P(A \cap B \cap C) = 1/4$

$P(A) \cdot P(B) \cdot P(C) = \frac{1}{8}$

So A, B, C not indep.

A = first card is ♡ $\qquad P(A) = \frac{13}{52} = P(B)$

B = second card is ♡

$$P(A \cap B) = \frac{\binom{13}{2}}{\binom{52}{2}} = \frac{\frac{13 \cdot 12}{2}}{\frac{52 \cdot 51}{2}} = \frac{13}{52} \cdot \frac{12}{51} \neq \left(\frac{13}{52}\right)^2$$

$$P(A \cap B) = P(A) \cdot P(B|A) = \frac{13}{52} \cdot \frac{12}{51}$$

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note: $P(A \cap B \cap C \cap D) = P(A) \cdot P(B|A) \cdot P(C|A \cap B) \cdot P(D|A \cap B \cap C)$

if indep: $\qquad = P(A) \cdot P(B) \cdot P(C) \cdot P(D)$