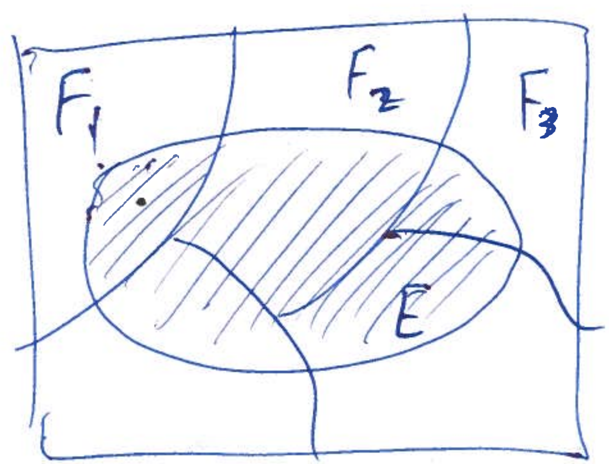
2023-01-18

Two dice: consider only total showing.

$P(\{2\}) = \frac{1}{36}$ $\quad$ $P(\{7\}) = \frac{6}{36}$ $\quad$ $P(\{2,7\}) = \frac{7}{36}$

If all outcomes equally likely, $P(E) = \frac{|E|}{|S|}$



__Thm__ Let $\{F_1, F_2 \ldots\}$ be a partition of S

E any event.

(a) $P(E) = \sum_i P(E|F_i) \cdot P(F_i)$ Law of total prob.

(b) $P(F_i|E) = \dfrac{P(E|F_i) \cdot P(F_i)}{\sum_j P(E|F_j)\, P(F_j)}$ Bayes Formula.

__partition__ : disjoint events with $S = \bigcup_{i=1}^{n} F_i$

__Proof__ E is a disjoint union of $E \cap F_i$

so $P(E) = \sum_i P(E \cap F_i) = \sum_i P(F_i) \cdot P(E|F_i)$

(b) $P(F_i|E) = \dfrac{P(E\cap F_i)}{P(E)} = \dfrac{P(E|F_i)\cdot P(F_i)}{\sum_j P(E|F_j)\cdot P(F_j)}$ $\square$

Pick unif. 1...10, throw that many dice.

E = sum is 3.

Let F_i = pick i. $\quad P(F_i) = \frac{1}{10}$

$$\left.\begin{aligned} P(E|F_1) &= \tfrac{1}{6} \\ P(E|F_2) &= \tfrac{2}{36} \\ P(E|F_3) &= \tfrac{1}{6^3} \\ P(E|F_k) &= 0 \text{ for } k>3 \end{aligned}\right\} \quad P(E) = \frac{1}{10}\cdot\frac{1}{6} + \frac{1}{10}\cdot\frac{2}{36} + \frac{1}{10}\cdot\frac{1}{6^3} + 0 + 0 \ldots$$

Prob that picked 2 | sum = 3 :

$$P(F_2 | E) = \frac{\frac{1}{10} \cdot \frac{2}{36}}{\frac{1}{10} \cdot \frac{1}{6} + \frac{1}{10} \cdot \frac{2}{36} + \frac{1}{10} \cdot \frac{1}{6^3}}$$

---

medical test for a disease.

False positive : result is Yes, person is healthy.

False negative : " No " " sick.

Assume $P(\text{False neg.}) = \delta = P(\text{result is No} \mid \text{sick person})$

$P(\text{False pos}) = \varepsilon = P(\text{result is Yes} \mid \text{health person})$.

Assume that $\frac{1}{100}$ people is infected.

$\delta = \frac{1}{100}$ $\quad \varepsilon = \frac{5}{100}$

If a test is pos. what is Prob. that person is infected

F_1 = healthy $\qquad F_2$ = sick $\qquad E$ = test pos.

$P(F_1) = \frac{99}{100} \qquad P(F_2) = \frac{1}{100}$

$P(E|F_1) = \varepsilon = \frac{5}{100} \qquad P(E|F_2) = 1-\delta = \frac{99}{100}$

$$P(F_2|E) = \frac{P(E|F_2)\cdot P(F_2)}{P(E|F_1)\,P(F_1) + P(E|F_2)\,P(F_2)} =$$

$$= \frac{99/100 \cdot 1/100}{5/100 \; \frac{99}{100} \quad \frac{99}{100}\cdot 1/100} = \frac{99}{5\cdot 99 + 99} = \frac{1}{6}$$

Doors 1, 2, 3. One has prize.

Contestant picks one of them.

Monty Hall reveals that another is wrong.

Should player switch?

Assume player picked 1.

F_i = prize is in i

Host reveals door 2 has no prize : E

$P(F_1 | E) = ?$

$P(F_3 | E) = ?$

$$P(F_1) = P(F_2) = P(F_3) = \frac{1}{3}$$

$$P(E|F_2) = 0 \qquad P(E|F_3) = 1 \qquad P(E|F_1) = \frac{1}{2}$$

Bayes : $$P(F_1|E) = \frac{P(E|F_1)\cdot P(F_1)}{\sum P(E|F_j)\, P(F_j)} = \frac{\frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3} + 0\cdot\frac{1}{3} + 1\cdot\frac{1}{3}}$$

$$= \frac{1}{3}$$

$$P(F_3|E) = \frac{1\cdot\frac{1}{3}}{\frac{1}{2}\cdot\frac{1}{3} + 0\cdot\frac{1}{3} + 1\cdot\frac{1}{3}} = \frac{2}{3}$$