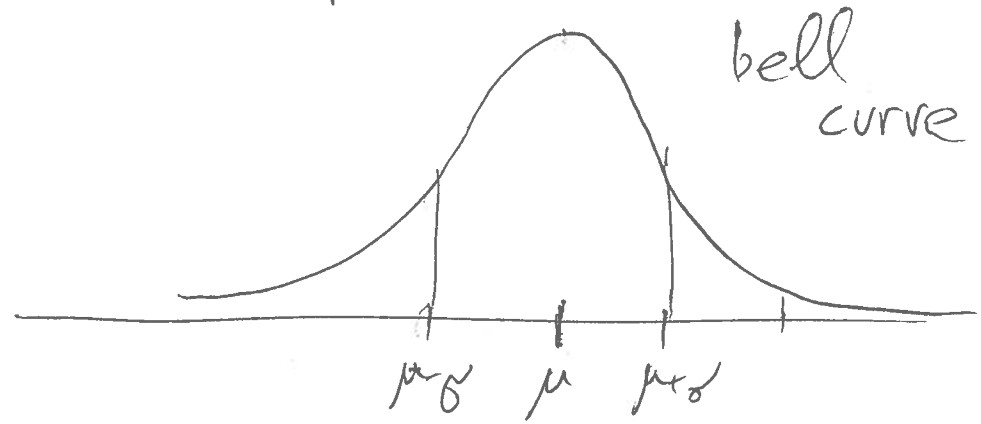
$\int_{-\infty}$

Gaussian / Normal R.V.

pdf of a contin. R.V. X is f s.t.

~~N(σ~~ $N(\mu, \sigma^2)$ is a R.V. with pdf

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Claim: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

change to polar coord:

Proof:

$$I = \int_0^{\infty}\int_0^{2\pi} e^{-r^2/2\sigma^2}\, r\, d\theta\, dr = 2\pi \int_0^{\infty} e^{-r^2/2\sigma^2}\, r\, dr$$

cov: $t = \frac{r^2}{2\sigma^2}$

$dt = \frac{2r}{2\sigma^2} dr$

$$= 2\pi \int_0^{\infty} e^{-t} \sigma^2\, dt = 2\pi\sigma^2$$

$I^2 = 2\pi\sigma^2$ so $I = \sqrt{2\pi}\,\sigma$ $\square$

CDF : $\Phi(x) = P(X \le x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$

version erf
error func

other 

 σ small

$\mathrm{erf}(x) = \int_0^x e^{-z^2} \, dz$

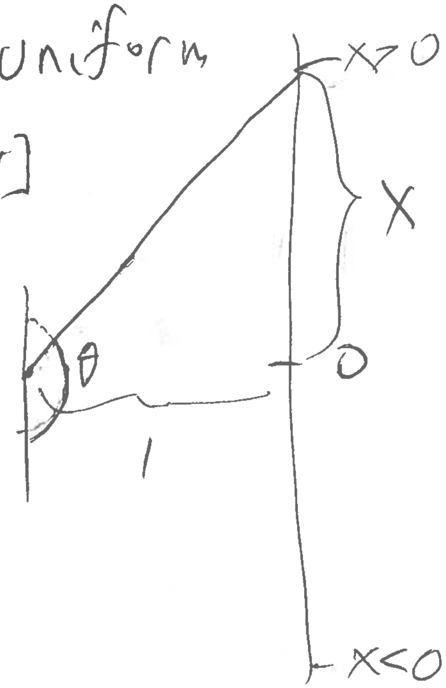
Scaling of normal variables: If X is $N(\mu, \sigma^2)$ and $Y = \frac{X-\mu}{\sigma}$ then Y is $N(0,1)$

$X = \sigma \cdot Y + \mu$

cov : $z = \frac{x-\mu}{\sigma}$ converts this to (*) $\square$

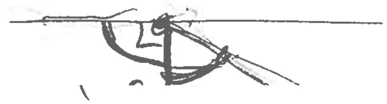
Pf: we show $P(Y \le t) = \int^{t} \frac{1}{\;} e^{-z^2/2}\, d$ ~~(*)~~

Θ is uniform in $[0,\pi]$

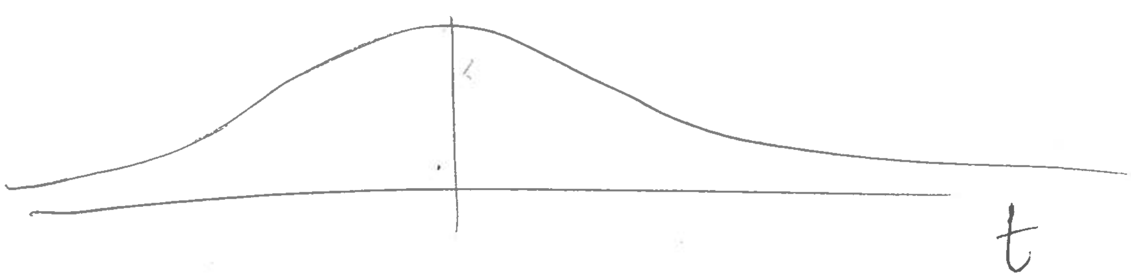


Qn find dist. of X.

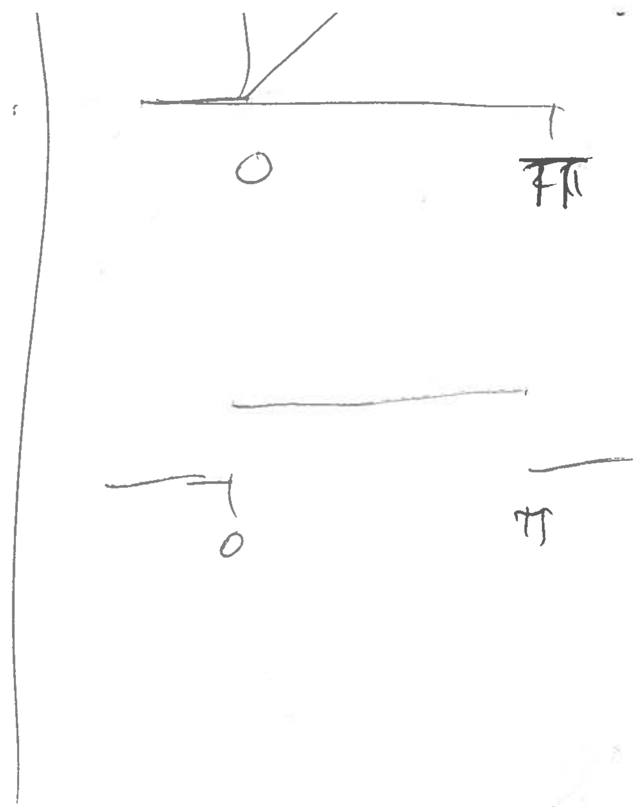
$$P(X \le t) = \frac{1}{\pi}\left(\tan^{-1} t + \frac{\pi}{2}\right) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1} t$$



$$f(t) = \frac{d}{dt}\left(P(X \le t)\right) = \frac{1}{\pi}\frac{1}{1+t^2}$$



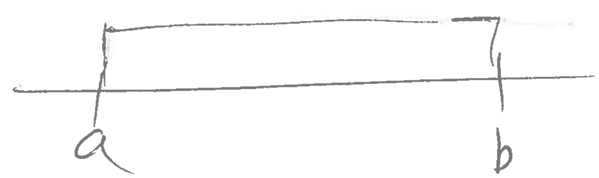
Cauchy R.V.



Expectation: (c~~ts~~ RC):

eg. Uniform $[a,b]$

$$f = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{if not} \end{cases}$$



$EX = \frac{a+b}{2}$

$$\int_{-\infty}^{\infty} x f(x)\, dx = \cancel{\int_a^b x \frac{b-a}{2} dx = \frac{a+b}{2}}$$

$$= \int_a^b \frac{x}{b-a} dx = \frac{a+b}{2}$$

$E N(\mu, \sigma^2) = \mu$

e.g. X is $\mathrm{Exp}(\lambda)$: $f = \lambda e^{-\lambda x}$ for $x \geq 0$.

EX for X Cauchy : $\int_{-\infty}^{\infty} \frac{x}{\pi(1+x^2)} dx$ undefined