Recall

Sample mean $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ $\quad$ $X_1 \dots X_n$ are iid

Sample Variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

Hypothesis testing: If (likelyhood of data if hyp. is true) is small, reject hypothesis

depends on tolerance for error. 5%

If 20 people run experiments then on avg. one will get statically significant result by chance.

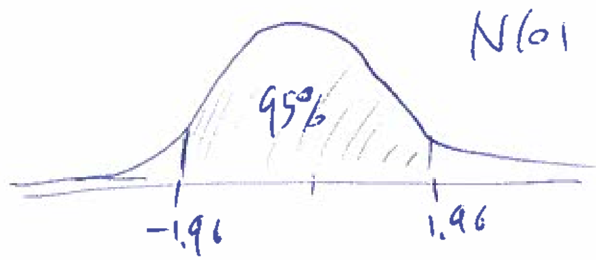
e.g. measure speed of light

Yesterday it was $300 \cdot (10^6 \, m/s)$

Assume that if speed is μ, measurements are $N(\mu, 5^2)$

If single X_1 is $|X_1 - 300| \geq 1.96 \cdot 5$ reject hyp that $\mu = 300$.

1.96 since



given 10 samples: reject hyp. if $|\bar{X} - 300| \geq a$ where

a chosen s.t. $P(|\bar{X} - 300| \geq a \mid \text{null-hyp}) = 0.05$

To find a: under null-hyp. $\bar{X} \sim N(300, \frac{5^2}{10})$

$$Var(\textstyle\sum X_i) = \sum Var(X_i) = 10 \cdot 5^2$$

$a = 1.96 \cdot \sqrt{Var(\bar{x})} = 1.96 \cdot \sqrt{2.5} \approx 3.1$

so if $|\bar{X} - 300| \geq 3.1$ reject null hyp. with 95% confidence.

95% confidence interval : Those values that we can not reject with 95% confidence.

e.g. for 10 measurements $N(\mu, 5^2)$ the 95% C.I. is $[\bar{X} - 3.1, \bar{X} + 3.1]$

Qn : what if $Var(X_i)$ unknown ?

e.g. $(X_i) = \{294, 301, 303, 296, 291\}$

$(X_i) = \{296, 297, 298, 297, 297\}$

__Idea__: If X_i are $N(\mu, \sigma^2)$ μ, σ unknown, can use S^2 to estimate σ^2.

Reject Hyp. that $\mu = \mu_0$ if $\left|\frac{\bar{X} - \mu_0}{S/\sqrt{n}}\right| \geq 1.96$

for 95% conf.

[CLT: $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \approx N(0,1)$]

If n large, $S \approx \sigma$ so $\frac{\bar{X} - \mu}{S/\sqrt{n}}$ is $\approx N(0,1)$.

For small n, $\frac{\bar{X} - \mu}{S/\sqrt{n}}$ has Student-T distribution,

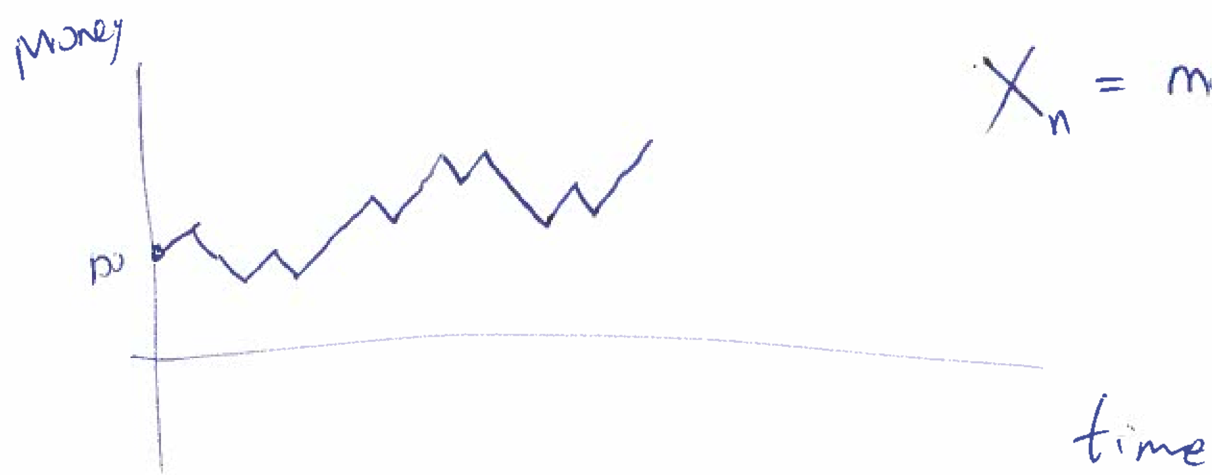
pdf $f_n(t) = \frac{c}{\left(1 + \frac{t^2}{n}\right)^{\frac{n+1}{2}}}$

Markov chains:

e.g. Gambler enters a casino with 100.

Gamble 1 each time.
$$\begin{cases} \text{win } 1 & \text{w.p. } \frac{18}{38} \\ \text{lose } 1 & \text{w.p. } \frac{20}{38} \end{cases}$$

Gary stops when either have 0 or 500.



X_n = money after n bets.

$P(\text{get to } 500) = ?$