

Mathematics 318 — Sample midterm

1. Let X have c.d.f. $F(t) = \begin{cases} 0 & t < 0, \\ \sin(t/2) & t \in [0, \pi], \\ 1 & t > \pi. \end{cases}$
 - (a) What is $E[X]$?
 - (b) What are the CDF and PDF of $Y = \sqrt{X}$?
2. Let X, Y be random variables with joint p.d.f. $f(x, y) = x + y$ on the square $[0, 1] \times [0, 1]$ and 0 outside.
 - (a) Are X and Y independent?
 - (b) What is the marginal distribution of X ?
 - (c) What is the Covariance (X, Y) ?
3. Let X, Y be uniform on the square $[0, 2] \times [0, 2]$. Find the distribution of the ratio $Z = X/Y$.
4. Let (X, Y) be the result of rolling two (6-sided) dice.
 - (a) What is the characteristic function of X ?
 - (b) What is the characteristic function of $X + Y$?
 - (c) Are the events A, B, C independent, where
$$A = \{X \text{ is even}\}, \quad B = \{y \text{ is even}\}, \quad C = \{X + Y = 5\}.$$
 - (d) Which pairs out of A, B, C are independent?
5. An urn contains 4 red, 6 green, and 10 blue balls.
 - (a) If three balls are drawn **with** replacement, what is the probability that the balls have three different colours?
 - (b) If three balls are drawn **without** replacement, what is the probability that they are all the same colour?
 - (c) If three balls are drawn **without** replacement, what is the probability that they are all blue conditioned on the event that they all have the same colour?

Table 1: Common Distributions

Distribution	Mean	Variance	Characteristic function
Bin(n, p)	np	$np(1 - p)$	$(1 - p + pe^{it})^n$
Geom(p)	$1/p$	$\frac{1 - p}{p^2}$	$\frac{pe^{it}}{1 - (1 - p)e^{it}}$
Poi(λ)	λ	λ	$e^{\lambda(e^{it} - 1)}$
Unif[a, b]	$\frac{a + b}{2}$	$\frac{(b - a)^2}{12}$	$\frac{e^{ita} - e^{itb}}{it(b - a)}$
Exp(λ)	$1/\lambda$	$1/\lambda^2$	$\frac{\lambda}{\lambda - it}$
N(μ, σ^2)	μ	σ^2	$e^{i\mu t - \sigma^2 t^2 / 2}$

The normal CDF:

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990