Note: This assignment should be typeset. Make sure your name and student number are on the top of page 1 (or all pages).

Problem 1. What is a game? Give examples of games with and without randomness, and with or without possible ties (all 4 combinations).

Problem 2. Alice and Bob play tic-tac-toe. Alice goes first. Consider the following strategy $S$:
- If I can win right now, play to win.
- If not, and the opponent is about to win, block them.
- Otherwise, play in the center if possible.
- Otherwise, play in a corner if possible (first available, top to bottom, left to right).
- Otherwise play in a side square (same order of preference).

Questions:
(a) If both players use strategy $S$, what is the outcome of the game? Draw the resulting board.
(b) If only Bob (as player 2) uses strategy $S$, show that Alice has a strategy that wins.

Problem 3. Recall the game of CHOMP from class (see Chapter 1 of Karlin+Peres).
(a) How many possible positions might come up during the game starting with a $3 \times 3$ board? Include the starting and final position.
(b) [bonus] How many possible position in the game started with an $n \times m$ board?

Problem 4. Alice and Bob play CHOMP, starting with a $2 \times 4$ board. Alice goes first.
(a) Alice's strategy is to always take a single square from the top row if possible, or from the bottom if the top row is all gone. Is this a winning strategy? If not, give a strategy for Bob that wins against this.
(b) What if Bob goes first? (with Alice still using the same strategy.)
(c) What happens if both players use this strategy?
(bonus) Describe a winning strategy for the first player.