Problem 1. Consider a game played on the following graph. An arrow from \( x \) to \( y \) means it is allowed to move from \( y \) to \( x \). (The same graph is shown twice.)
(a) Find the P and N positions.
(b) If the game starts at position \( a \), who wins (first or second player)?
(c) Find the Sprague-Grundy value of each state.

![Graph](image)

Problem 2. A chess queen is placed on a chess board, but is allowed to move only left, down, or diagonally left-down. (There is no move from the bottom left corner.) Find the Sprague-Grundy value of each position in a \( 5 \times 5 \) board.

Problem 3. Consider the subtraction game with subtraction set \( \{2, 3, 4, 5\} \).
(a) Find the Sprague-Grundy value for piles of size up to 13.
(b) If there are piles of sizes \( \{6, 7, 8, 9\} \), (and we are allowed to remove 2,3,4, or 5 chips from one of the piles), find a winning move.
(c) For the same position, find all winning moves.
(d) What is \( g(344) \)?

Problem 4. (a) Find all optimal strategies for each player in the zero-sum game with matrix \( A \) below.
(b) Find some optimal strategy for each player in the zero-sum game with matrix \( B \) below.

\[
A = \begin{pmatrix} 0 & 2 & 3 & 5 & 8 \\ 8 & 5 & 3 & 2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 6 & 5 & 2 & 1 \\ 5 & 4 & 3 & 0 \\ 1 & 0 & 5 & 4 \\ 2 & 0 & 2 & 3 \end{pmatrix}
\]

Problem 5. Find the value and an optimal strategy for the game \( A = \begin{pmatrix} 0 & 6 \\ 4 & s \end{pmatrix} \) for an arbitrary value \( s \). Draw a graph of the value as a function of \( s \) (from \(-\infty\) to \(\infty\)).

Problem 6. Find a hyperplane which separates 0 from the set \( K \in \mathbb{R}^3 \) given by \( K = \{xyz \geq 1, x, y, z \geq 0\} \).