Problem 1. The Total variation distance $d_{TV}(X,Y)$ is defined as $\inf \mathbb{P}(X' \neq Y')$ over all couplings of X, Y. How does convergence in the total variation distance relate to the other notions of convergence discussed in class? (Including distribution, probability, a.s., L^1.)

Problem 2. Let X_n be i.i.d. uniform in $[0,1]$ random variables. Let $Y_n = 1$ if X_n is a local maximum: $X_n = \max\{X_n, X_{n-1}, X_{n+1}$. Let $S_n = \sum_{i=1}^n Y_i$. Find constants a, b so that the sum of Y_n has a CLT:

$$\frac{S_n - an}{b\sqrt{n}} \to N(0,1),$$

and prove this convergence. (Hint: The Y_n are not independent, but only depndent for nearby n's. Approximate S_n by a sum of independent R.V.s.)

Problem 3. Prove that φ_X is periodic if and only if X takes values in $a\mathbb{Z}$ (multiples of a) for some a.

Problem 4. Write a proof of the following statement from class: If $X \geq 0$ is a R.V. then $\mathbb{E}X = \int_0^\infty \mathbb{P}(X > t)dt$. Moreover $\mathbb{E}X^a = \int_0^\infty t^{a-1}\mathbb{P}(X > t)dt$.

Problem 5. Prove that for any random variable with finite mean, $t\mathbb{P}(X > t) \to 0$ as $t \to \infty$. Moreover, if $\mathbb{E}|X|^a < \infty$ then $t^a\mathbb{P}(X > t) \to 0$.

Problem 6. If $X_n \xrightarrow{prob} X$ are independent, show that X must be a constant random variable.

Problem 7. Show that convergence in probability is equivalent to convergence in the metric

$$d(X,Y) = \mathbb{E}(|X - Y| \wedge 1).$$