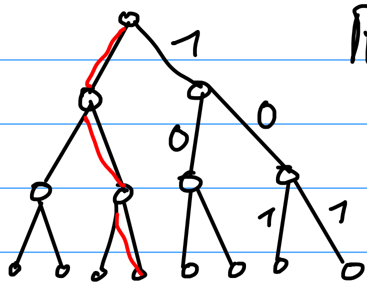
Branching random walks, continuation

BRW with killing/selection/interactions

Example for a BRW with killing: $p(2) = 1$



$\mathbb{P}[X = 0] = 1 - p$

$\mathbb{P}[X = 1] = p$

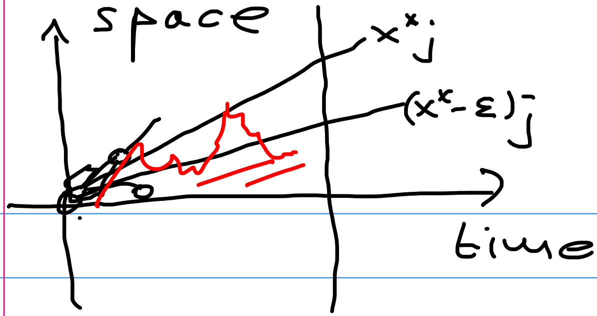
ray: $v_0, v_1, v_2, v_3, \ldots$
v_{i+1} child of v_i

$$\rho(\varepsilon, p) = \mathbb{P}[\exists \text{ inf. ray s.t. } S_{v_j} \geq (x^* - \varepsilon) j, \ \forall j \geq 1]$$

$$= \mathbb{P}[\text{"nearly optimal ray"}]$$

<u>Q</u> $\rho(\varepsilon, p) \sim$? (for fixed p, as $\varepsilon \to 0$)

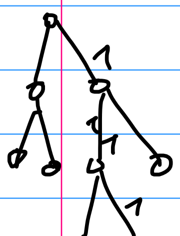
- For $p > \frac{1}{2}$, $x^* = 1$, with pos. prob. there is an optimal ray $\Rightarrow \rho(\varepsilon, p) \underset{\varepsilon \to 0}{\not\to} 0$



- $p = \frac{1}{2}$ $\quad \rho(\varepsilon, p) \sim c\,\varepsilon$
(R. Pemantle)

Take $\left(\varepsilon = \frac{1}{n}\right)$ $\quad n = \frac{1}{\varepsilon}$

$\mathbb{P}_{\frac{1}{2}}[\,\exists$ ray only 1's up to level $n\,]$

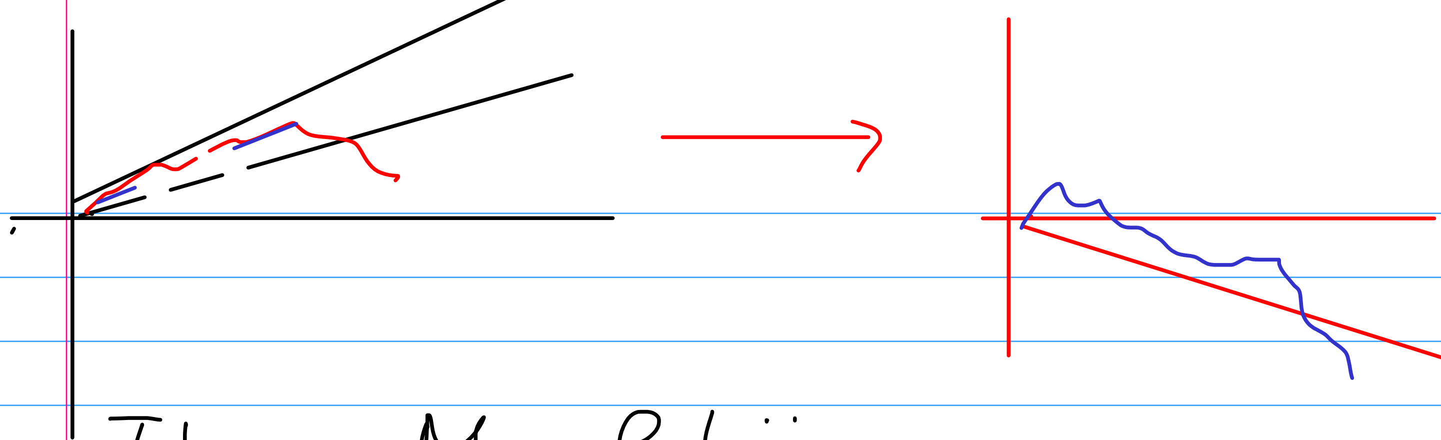


$\sim \frac{2}{n}$

($\mathbb{P}[$ crit. GW process survives up to gen. $n\,] \sim \frac{2}{n}$)

- For $p < \frac{1}{2}$,
$\log \rho(\varepsilon, p) \sim \frac{-c(p)}{\sqrt{\varepsilon}}$ for $\varepsilon \to 0$
(NG, Y. Hu, Z. Shi)

Idea:
Replace large deviations for endpoint with large deviations for the whole path.



Thm Mogulskii

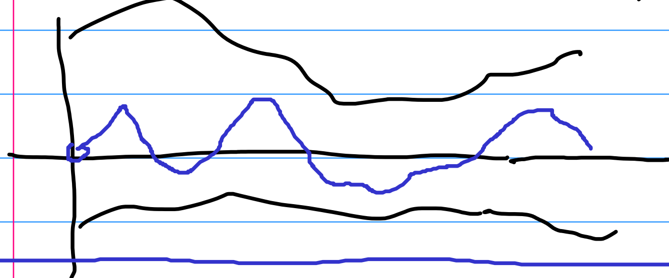
$Y_1, Y_2, \ldots$ iid $E[Y_i]=0$, $E|Y_1|^{2+\delta}]<\infty$

$S_n = \sum_{i=1}^{n} Y_i$ $\quad g_1(\cdot), g_2(\cdot)$ cont.

$$g_1(0) < 0 < g_2(0)$$
$$g_1 < g_2$$

$$\frac{a_n^2}{2} \log P\left[+g_1\left(\tfrac{i}{n}\right) \le \frac{S_i}{a_n} \le g_2\left(\tfrac{i}{n}\right),\ 1 \le i \le n\right] \longrightarrow -J(g_1, g_2, \sigma^2)$$

Here $a_n \longrightarrow \infty$, $\dfrac{a_n}{\sqrt{n}} \longrightarrow 0$

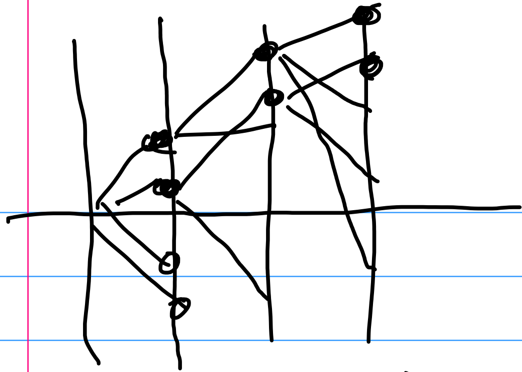


N-BRW

Example with selection

- keep only the N individuals with the largest positions

Brunet/Derrida/Mueller/Meunier



$$M_{n,N} = \max_{v \in D_{n,N}} S_v$$

$$\boxed{x_N^* \leq x^*}$$

Then $\dfrac{M_{n,N}}{n} \xrightarrow[n\to\infty]{} x_N^*$ a.s.

Thm J. Bérard, J.-B. Gouéré

$p(2) = 1$, $\mathbb{E}[e^{\lambda X}] < \infty \quad \forall \lambda \in \mathbb{R}$

$$x^* - x_N^* \sim \frac{\text{const}}{(\log N)^2} \text{ for } N \to \infty$$

$\longrightarrow$ J. Bérard P. Maillard for heavy-tailed X, i.e

$$\mathbb{P}[X > t] \sim \frac{1}{t^\alpha} \qquad 0 < \alpha < 2$$

Heuristics: The foll. two events are comparable:

(1) BRW with killing at slope $x^* - \varepsilon$, starting with N particles, survives

(2) N - BRW moves at a speed $\geq x^* - \varepsilon$.

(1) Prob. is $1 - (1 - \rho(\varepsilon))^N$

$\Rightarrow$ (2) $x^* - x_N^*$ should be of order $\varepsilon = \varepsilon(N)$ with ε s.t.

$$\rho(\varepsilon) \approx \frac{1}{N}$$

But since $\rho(\varepsilon) \approx e^{-\frac{c}{\sqrt{\varepsilon}}} \approx \frac{1}{N}$ have $\varepsilon \approx \frac{c}{(\log N)^2}$.

L-BRW

- keep only the individuals within (spatial) distance L to M_n, remove all the others.

$$x_L^* = \lim_{n\to\infty} \frac{M_{n,L}}{n}$$

Clear: $x_L^* \le x^*$

Conj BDMM

$x_L^* \approx x^*_{L=\log N}$

$$\boxed{x^* - x_L^* = \frac{c}{L^2}}$$

Thm for L-BBM by M. Pain.

BRW with interactions

Q Take a BRW with some $X \overset{d}{=} N(0,1)$. Fix $R \in \mathbb{R}$

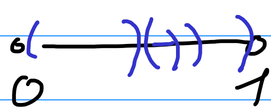
- kill two particles if they come closer than R.

Q Is the survival probability strictly positive? Open!

Fragmentation process

$(I_t)_{t \geq 0}$

Fix $\alpha > 0$.



I_t is collection of disjoint interval $\subseteq (0,1)$

An interval (a,b) with $u = b-a$ splits* into m subintervals $(a, a+\frac{u}{m}, a+\frac{2u}{m}, \dots, b)$

*at rate u^α

$N_t(j) = \#$ intervals (a,b) at time t s.t. $b-a = m^{-j}$

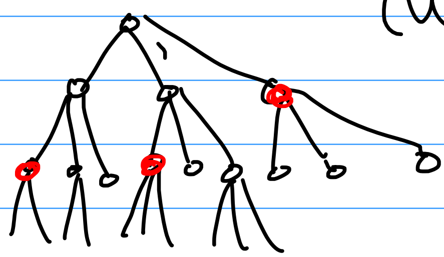
- Multitype branching process

Types in $\mathbb{N}_0$. Particles die at rate q^j, replaced by m offspring of type $j+1$.

$(q > 0$ fixed$)$

$N_t(j)$ = # particles of type j present at time t

- **Tree-indexed RW**



(W_e) indep.

$W_e \overset{d}{=} \mathrm{Exp}(q^n)$ if e goes from level $n-1$ to level n.

Say v is alive at time t if $S_v \le t$, $S_{v_i} > t$ $\forall\, 1 \le i \le m$, v_i child of v

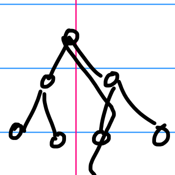
Study $N_t = \sum_{n \ge 0} N_t(n)$

- Work in progress with P. Duszewski, S. Johnston, J. Prochno, D. Schmid

Thm Brennan / Durrett

$q<1$: $E[N_t] \sim \dfrac{t^{\beta}}{} = t^{\frac{1}{\alpha}}$ where $\beta = \dfrac{\log m}{\log \frac{1}{q}}$

$q = m^{-\alpha}$



$$E[N_t] = \sum_{\ell=0}^{\infty} m^{\ell} e^{-q^{\ell} t}$$

- $q > 1$ "explosive"
- $q = 1$ "classical"
- $q < 1$ "slow"