Mean field asymptotics in high-dimensional statistics: A few references

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Abstract

This is a guided bibliography to some theoretical topics in high-dimensional statistics and probability theory that are covered during the OOPS summer school in July 2020. This list of references is incomplete even for what concerns this set of topics. I will be improving it.

1 Background material

Statistics [BVDG11]. Physics and algorithms [EVdB01, MM09].

2 Exact asymptotics

Various approaches. Early approaches in the context of compressed sensing made use of tools from convex geometry [DT10b, DT10a], which were substantially refined in [ALMT14]. A sharp asymptotic characterization od the Lasso was first obtained in [BM12] using an analysis via AMP. Other papers that use the same approach include [DM16, CS18, SC19],

Leave-one out techniques were used in [EKBB+13, EK18].

Gaussian comparison. Gordon inequality was first proven in [Gor88]. Its application to convex-concave problems developed in [TOH15]. Applications of this approach include [TAH18, MM18, SAH19].

Bayes optimal estimators. Exact asymptotics for the Bayes error were derived in [DAM16, BDM+16], using again the connection to AMP, in [LM19, Mio17] using leave-one-out techniques. Adaptive interpolation method [BM19, BKM+19].

3 Approximate Message Passing

'Historical' background on AMP and its motivations can be found in [TAP77, Kab03, DMM09].

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Sharp analysis of AMP algorithms was developed in various degrees of generality, beginning with [Bol14] and then in [BM11, BLM15, JM13, BMN20, CL20]. (In particular [BMN20] streamlines and generalized the conditioning proof.) Optimality of Bayes-AMP among generalized first order methods was proven in [CMW20].

4 Optimization of mean-field spin glasses

The classical physics papers in this area are collected in [MPV87]. For a survey of mathematical work in this area, see [Tal10, Pan13].

Important structural properties of Parisi formula were proven in [JT16, AC17, Che17, AC15].

Optimization algorithms for mean field spin glasses were developed in [Sub18] (for the spherical case) and [Mon19, AMS20] (for the Ising case).

Negative results about optimization in problems *with* overlap gap were proven among others in [GS14, GJ19, GJW20].

References

[AC15] Antonio Auffinger and Wei-Kuo Chen, *The Parisi formula has a unique minimizer*, Communications in Mathematical Physics **335** (2015), no. 3, 1429–1444. 2

[AC17] ———, *Parisi formula for the ground state energy in the mixed p-spin model*, The Annals of Probability **45** (2017), no. 6b, 4617–4631. 2

[ALMT14] Dennis Amelunxen, Martin Lotz, Michael B McCoy, and Joel A Tropp, *Living on the edge: Phase transitions in convex programs with random data*, Information and Inference: A Journal of the IMA **3** (2014), no. 3, 224–294. 1

[AMS20] Ahmed El Alaoui, Andrea Montanari, and Mark Sellke, *Optimization of mean-field spin glasses*, arXiv:2001.00904 (2020). 2

[BDM+16] Jean Barbier, Mohamad Dia, Nicolas Macris, Florent Krzakala, Thibault Lesieur, and Lenka Zdeborová, *Mutual information for symmetric rank-one matrix estimation: A proof of the replica formula*, Advances in Neural Information Processing Systems, 2016, pp. 424–432. 1

[BKM+19] Jean Barbier, Florent Krzakala, Nicolas Macris, Léo Miolane, and Lenka Zdeborová, *Optimal errors and phase transitions in high-dimensional generalized linear models*, Proceedings of the National Academy of Sciences **116** (2019), no. 12, 5451–5460. 1

[BLM15] Mohsen Bayati, Marc Lelarge, and Andrea Montanari, *Universality in polytope phase transitions and message passing algorithms*, The Annals of Applied Probability **25** (2015), no. 2, 753–822. 2

[BM11] Mohsen Bayati and Andrea Montanari, *The dynamics of message passing on dense graphs, with applications to compressed sensing*, IEEE Trans. on Inform. Theory **57** (2011), 764–785. 2

[BM12] _____, *The LASSO risk for gaussian matrices*, IEEE Trans. on Inform. Theory **58** (2012), 1997–2017. 1

[BM19] Jean Barbier and Nicolas Macris, *The adaptive interpolation method: a simple scheme to prove replica formulas in bayesian inference*, Probability Theory and Related Fields **174** (2019), no. 3-4, 1133–1185. 1

[BMN20] Raphael Berthier, Andrea Montanari, and Phan-Minh Nguyen, *State evolution for approximate message passing with non-separable functions*, Information and Inference: A Journal of the IMA **9** (2020), no. 1, 33–79. 2

[Bol14] Erwin Bolthausen, *An iterative construction of solutions of the TAP equations for the Sherrington–Kirkpatrick model*, Communications in Mathematical Physics **325** (2014), no. 1, 333–366. 2

[BVDG11] Peter Bühlmann and Sara Van De Geer, *Statistics for high-dimensional data: methods, theory and applications*, Springer Science & Business Media, 2011. 1

[Che17] Wei-Kuo Chen, *Variational representations for the Parisi functional and the two-dimensional Guerra–Talagrand bound*, The Annals of Probability **45** (2017), no. 6A, 3929–3966. 2

[CL20] Wei-Kuo Chen and Wai-Kit Lam, *Universality of approximate message passing algorithms*, arXiv preprint arXiv:2003.10431 (2020). 2

[CMW20] Michael Celentano, Andrea Montanari, and Yuchen Wu, *The estimation error of general first order methods*, arXiv:2002.12903 (2020). 2

[CS18] Emmanuel J Candès and Pragya Sur, *The phase transition for the existence of the maximum likelihood estimate in high-dimensional logistic regression*, arXiv:1804.09753 (2018). 1

[DAM16] Yash Deshpande, Emmanuel Abbe, and Andrea Montanari, *Asymptotic mutual information for the balanced binary stochastic block model*, Information and Inference: A Journal of the IMA **6** (2016), no. 2, 125–170. 1

[DM16] David Donoho and Andrea Montanari, *High dimensional robust m-estimation: Asymptotic variance via approximate message passing*, Probability Theory and Related Fields **166** (2016), no. 3-4, 935–969. 1

[DMM09] David L. Donoho, Arian Maleki, and Andrea Montanari, *Message Passing Algorithms for Compressed Sensing*, Proceedings of the National Academy of Sciences **106** (2009), 18914–18919. 1

[DT10a] D. L. Donoho and J. Tanner, *Counting the faces of randomly-projected hypercubes and orthants, with applications*, Discrete & Computational Geometry **43** (2010), no. 3, 522–541. 1

[DT10b] D.L. Donoho and J. Tanner, *Exponential bounds implying construction of compressed sensing matrices, error-correcting codes, and neighborly polytopes by random sampling*, IEEE Trans. on Inform. Theory **56** (2010), no. 4, 2002–2016. 1

[EK18] Noureddine El Karoui, *On the impact of predictor geometry on the performance on high-dimensional ridge-regularized generalized robust regression estimators*, Probability Theory and Related Fields **170** (2018), no. 1-2, 95–175. 1

[EKBB+13] Noureddine El Karoui, Derek Bean, Peter J Bickel, Chinghway Lim, and Bin Yu, *On robust regression with high-dimensional predictors*, Proceedings of the National Academy of Sciences **110** (2013), no. 36, 14557–14562. 1

[EVdB01] Andreas Engel and Christian Van den Broeck, *Statistical mechanics of learning*, Cambridge University Press, 2001. 1

[GJ19] David Gamarnik and Aukosh Jagannath, *The overlap gap property and approximate message passing algorithms for p-spin models*, arXiv preprint arXiv:1911.06943 (2019). 2

[GJW20] David Gamarnik, Aukosh Jagannath, and Alexander S Wein, *Low-degree hardness of random optimization problems*, arXiv preprint arXiv:2004.12063 (2020). 2

[Gor88] Yehoram Gordon, *On Milman's inequality and random subspaces which escape through a mesh in R^n*, Geometric Aspects of Functional Analysis, Springer, 1988, pp. 84–106. 1

[GS14] David Gamarnik and Madhu Sudan, *Limits of local algorithms over sparse random graphs*, Proceedings of the 5th conference on Innovations in theoretical computer science, ACM, 2014, pp. 369–376. 2

[JM13] Adel Javanmard and Andrea Montanari, *State evolution for general approximate message passing algorithms, with applications to spatial coupling*, Information and Inference: A Journal of the IMA **2** (2013), no. 2, 115–144. 2

[JT16] Aukosh Jagannath and Ian Tobasco, *A dynamic programming approach to the parisi functional*, Proceedings of the American Mathematical Society **144** (2016), no. 7, 3135–3150. 2

[Kab03] Yoshiyuki Kabashima, *A CDMA multiuser detection algorithm on the basis of belief propagation*, J. Phys. A **36** (2003), 11111–11121. 1

[LM19] Marc Lelarge and Léo Miolane, *Fundamental limits of symmetric low-rank matrix estimation*, Probability Theory and Related Fields **173** (2019), no. 3-4, 859–929. 1

[Mio17] Léo Miolane, *Fundamental limits of low-rank matrix estimation*, arXiv:1702.00473 (2017). 1

[MM09] Marc Mézard and Andrea Montanari, *Information, Physics and Computation*, Oxford, 2009. 1

[MM18] Léo Miolane and Andrea Montanari, *The distribution of the lasso: Uniform control over sparse balls and adaptive parameter tuning*, arXiv:1811.01212 (2018). 1

[Mon19] Andrea Montanari, *Optimization of the Sherrington-Kirkpatrick Hamiltonian*, IEEE Symposium on the Foundations of Computer Science, FOCS, November 2019. 2

[MPV87] Marc Mézard, Giorgio Parisi, and Miguel A. Virasoro, *Spin glass theory and beyond*, World Scientific, 1987. 2

[Pan13] Dmitry Panchenko, *The Sherrington-Kirkpatrick model*, Springer Science & Business Media, 2013. 2

[SAH19] Fariborz Salehi, Ehsan Abbasi, and Babak Hassibi, *The impact of regularization on high-dimensional logistic regression*, arXiv preprint arXiv:1906.03761 (2019). 1

[SC19] Pragya Sur and Emmanuel J Candès, *A modern maximum-likelihood theory for high-dimensional logistic regression*, Proceedings of the National Academy of Sciences **116** (2019), no. 29, 14516–14525. 1

[Sub18] Eliran Subag, *Following the ground-states of full-RSB spherical spin glasses*, arXiv:1812.04588 (2018). 2

[TAH18] Christos Thrampoulidis, Ehsan Abbasi, and Babak Hassibi, *Precise error analysis of regularized m-estimators in high dimensions*, IEEE Transactions on Information Theory **64** (2018), no. 8, 5592–5628. 1

[Tal10] Michel Talagrand, *Mean field models for spin glasses*, Springer-Verlag, Berlin, 2010. 2

[TAP77] David J. Thouless, Philip W. Anderson, and Richard G. Palmer, *Solution of 'solvable model of a spin glass'*, Philosophical Magazine **35** (1977), no. 3, 593–601. 1

[TOH15] Christos Thrampoulidis, Samet Oymak, and Babak Hassibi, *Regularized linear regression: A precise analysis of the estimation error*, Conference on Learning Theory, 2015, pp. 1683–1709. 1