Glossary

1. N-BBM Particles move in $\mathbb{R}$ as BMs, branch at rate 1, leftmost particle is killed.

$X^{(N)}(t) = (X_1^{(N)}(t), \dots, X_N^{(N)}(t))$ particle positions at time t.

$L_t^{(N)} = \min_{i \in \{1,\dots,N\}} X_i^{(N)}(t)$ leftmost particle position at time t.

$$
\text{(FBP1)} \begin{cases} \dfrac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u & \text{for } t>0,\ x > L_t \\ u(t, L_t) = 0 & \text{for } t>0 \\ \int_{L_t}^{\infty} u(t,y)\,dy = 1 & \text{for } t>0 \\ u(0,x) = u_0(x) & \text{for } x \in \mathbb{R}. \end{cases}
$$

$X^+(t) = (X_1^+(t), \dots, X_{N_t^+}^+(t))$ locations of particles in BBM at time t.

$H^+(t,x) := \frac{1}{N} \# \{ i \le N_t^+ : X_i^+(t) \geq x \}$

$H^{(N)}(t,x) := \frac{1}{N} \# \{ i \le N : X_i^{(N)}(t) \geq x \}$

$\mathcal{X} \succcurlyeq \mathcal{X}'$ iff $|\mathcal{X} \cap [x,\infty)| \geq |\mathcal{X}' \cap [x,\infty)|$ $\forall x \in \mathbb{R}$

$C_m f(x) = \min(f(x), m)$ "cut"

$G_t f(x) = \mathbb{E}_x[f(B_t)] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi t}} e^{-\frac{(x-y)^2}{2t}} f(y)\,dy$ "spread"

2. Brownian bees

$X^{(N)}(t) = (X_1^{(N)}(t), \dots, X_N^{(N)}(t))$ particle positions (in $\mathbb{R}^d$) at time t.

$M_t^{(N)} = \max_{i \in \{1,\dots,N\}} \| X_i^{(N)}(t) \|$ maximum particle distance from 0 at time t.

$\|\cdot\|$ is Euclidean (ℓ_2) norm

$$\text{(FBP2)}\quad\begin{cases} \dfrac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u & \|x\| < R_t,\ t>0 \\ u(t,x) = 0 & \|x\| \geq R_t,\ t>0 \\ \displaystyle\int_{\|x\|\leq R_t} u(t,x)\,dx = 1 & t>0 \\ u(t,x)\,dx \to \mu_0(dx) \text{ weakly as } t \searrow 0. \end{cases}$$

d-dim$^{\underline{\ell}}$ FBP

$B_r(x) := \{y \in \mathbb{R}^d : \|x-y\| < r\}.$

$\mu^{(N)}(t,dx) = \frac{1}{N}\sum_{k=1}^{N} \delta_{X_k^{(N)}(t)}(dx)$ empirical measure

$(u(x), R_\infty)$ steady state solution of (FBP2).

$F^{(N)}(t,r) := \mu^{(N)}(B_r(0),t) = \frac{1}{N}\#\{i \in \{1,\dots,N\} : \|X_i^{(N)}(t)\| < r\}.$

$$\text{(FBP3)}\quad\begin{cases} \dfrac{\partial v}{\partial t} = \frac{1}{2}\Delta v - \dfrac{d-1}{2r}\dfrac{\partial v}{\partial r} + v & t>0,\ r \in (0,R_t) \\ v(t,r) = 1 & t>0,\ r \geq R_t \\ \dfrac{\partial v}{\partial r}(t,R_t) = 0 & t>0 \\ v(t,0) = 0 & t>0 \\ v(0,\cdot) = v_0 & \end{cases}$$

one-dim$^{\underline{\ell}}$ FBP

$\mathbb{P}_{x^{(N)}}(\cdot) = \mathbb{P}(\cdot \mid X^{(N)}(0) = x^{(N)}).$

$(X_i^+(t),\ i \leq N_t^+)$ particle positions at time t in BBM.

$X_{i,t}^+(s) :=$ position of time-s ancestor of particle labelled i at time t.

Couple with Brownian bees so

$$X^{(N)}(t) = \{X_i^+(t) : i \leq N_t^+,\ \|X_{i,t}^+(s)\| \leq M_s^{(N)}\ \forall s \in [0,t]\}.$$

$V(r) = \int_{\|x\|<r} u(x)\,dx.$ $(V(x), R_\infty)$ is steady state solution of (FBP3).