MATH 184 Computing Derivatives

Some derivative rules which you can use mechanically are:

$$\text{SumRule}: \frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$
$$\text{Constant Multiple Rule}: \frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}f(x)$$
$$\text{Product Rule}: \frac{d}{dx}(f(x)g(x)) = \left(\frac{d}{dx}f(x)\right) \times g(x) + f(x) \times \left(\frac{d}{dx}g(x)\right)$$
$$\text{Quotient Rule}: \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{(\frac{d}{dx}f(x))g(x) - f(x)\frac{d}{dx}g(x)}{g^2(x)}$$
$$\text{Chain Rule}: \frac{d}{dx}f((g(x)) = \frac{d}{dx}f(x)|_{g(x)} \cdot \frac{d}{dx}g(x)$$

We are using the notation $h(x)|_a$ to denote the value of the function $h(x)$ at $x = a$. Another way to write the Chain Rule that may be easier is $(f(g(x))' = f'(g(x)) \cdot g'(x)$ where we have f' is $\frac{d}{dx}f(x)$. Thus the awkward expression $\frac{d}{dx}f(x)|_{g(x)}$ is the derivative of f evaluated at $g(x)$
(i.e. we substitute $g(x)$ for x). We cannot directly say that $\frac{d}{dx}x^0 = 0 \cdot x^{-1}$ since $\frac{d}{dx}x^0 = 0$ while $0 \cdot x^{-1} = 0$ for all $x \neq 0$ but is not defined for $x = 0$. This subtle difference will show up in MATH 105 when you wish to *integrate* (or *anti-differentiate*) the function $\frac{1}{x}$.

This will enable us to compute derivatives of quite complicated functions. Sometimes on tests we are only interested in *Calculator Ready* format which means that we computed a formula for the derivative but have not simplified.

We have some additional rules for special functions:

$$\tfrac{d}{dx}x^p = px^{p-1}(\text{for } p \neq 0)$$
$$\tfrac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln(x) = \frac{1}{x}$$
$$\tfrac{d}{dx}\sin(x) = \cos(x) \qquad \frac{d}{dx}\cos(x) = -\sin(x)$$

Try to puzzle out the following. We use a mixture of the rules. I think I have chosen mostly obvious rules so that for example $\frac{d}{dx}(g(x))^4 = 4 \cdot (g(x))^3 \cdot g'(x)$.

Examples

$$\frac{d}{dx}\left(\sin(x^2+5)(x^3+3x)^4\right) =$$
$$\left(-\cos(x^2+5) \cdot 2x\right)\left((x^3+3x)^4\right) + \left(\sin(x^2+5)\right) \cdot \left(4 \cdot (x^3+3x)^3 \cdot (3x^2+3)\right)$$

$$\frac{d}{dx}e^x \cdot \sin(x) \cdot \left(x^3 + x^{1.5}\right) =$$
$$e^x\left(\sin(x)\left(x^3+x^{1.5}\right)\right) + e^x\left((\cos(x)(x^3+x^{1.5}) + \sin(x)(3x^2+1.5x^{.5})\right)$$

These answers are in Calculator Ready form (all derivatives have been handled) but they are very unsimplified. It is not worth simplifying in such cases.