in my first summer job (in an office in 1973) having finished a linear algebra course I was asked to consider the problem of placing 100 points on the surface of a sphere (for a goniophotometer to measure light radiation from light bulbs). I determined the 100 points by an approximate process, namely simulating a repulsive force between the points and running it for a while until the closest distance between points had roughly stabilized. At that point I wanted to show the person who had asked the question, what my solution looked like.

I decided to take each of the 100 points and draw a circle of maximum radius around the point $P$ (on the surface of the sphere) by rotating the original point by an appropriate angle to begin the circle at a point $Q$ and then rotating $Q$ about the axis given by $OP$ by angles of $10^\circ$. We have the rotation matrix in 2-dimensions

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

We can extend to rotation in 3-dimensions about the $z$-axis:

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This isn’t what we want. We want rotation in 3-dimensions about an arbitrary axis $d = (d_1, d_2, d_3)^T$. So we employ a change of basis that replaces $(0, 0, 1)^T$ by $d/||d||$ and of course use an orthonormal basis $u, v, d$ for $\mathbb{R}^3$. We want a circle to remain a circle and not get squished into an ellipse. We can do this easily.

The matrix we want

$$M = \begin{bmatrix} u & v & d_1 \\ & & d_2 \\ & & d_3 \end{bmatrix}$$

will take the $u, v, d$ coordinates to standard coordinates. Then $M^{-1}$ takes standard coordinates to $u, v, d$ coordinates. We imagine $R$ as the rotation matrix with respect to $u, v, d$, namely rotation around $(0, 0, 1)^T$ in $u, v, d$ coordinates which is $d$ when interpreted in $u, v, d$ coordinates. Then we return from $u, v, d$ coordinates to standard coordinates using $M$. Thus $MRM^{-1}$ is our desired matrix. The rotation matrix around $d$ becomes

$$MRM^{-1} = M \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} M^{-1}$$

All my calculations had to be done on a machine that allowed me to store 400 memory locations including the code, so .4K. Space was so limited that I stored the 100 points in polar coordinates so they only needed 200 memory locations. To obtain the orthonormal basis I just found one vector perpendicular to $d$, then computed a vector orthogonal to both using the cross product formula.

The actual plotting was done using a mechanical plotter that could plot line segments using a felt tipped pen. These line segments were joining two points one rotated by a further $10^\circ$ using $MRM^{-1}$. You might well ask whether my points moved consistent with right hand rule about $d$ but that didn’t matter to me. The sign of the determinant of $R$ would have told me about the orientation. Of course I then had to finish using 3D projection. The date (1973) for all this is relevant.