1. Compute

\[ \begin{align*}
&i) \det \begin{bmatrix}
x & 0 & 0 \\
10 & x & 0 \\
52 & 223 & 1 \\
\end{bmatrix}, \\
&ii) \det \begin{bmatrix}
99 & 100 & 101 \\
0 & 0 & 0 \\
4 & e & 98 \\
\end{bmatrix}, \\
&iii) \det \begin{bmatrix}
2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2 \\
\end{bmatrix}
\end{align*} \]

2. (nice problem) Consider three 3 digit numbers write in decimal digits as \(\underbrace{abc}_{10}, \underbrace{def}_{10}\) and \(\underbrace{ghi}_{10}\). (e.g. \(abc_{10} = a \cdot 10^2 + b \cdot 10 + c\)). Assume that each of the numbers \(\underbrace{abc}_{10}, \underbrace{def}_{10}\) and \(\underbrace{ghi}_{10}\) is divisible by 17. Show that

\[ \begin{vmatrix}
a & d & g \\
b & e & h \\
c & f & i \\
\end{vmatrix} \]

is also divisible by 17.

3. Find the inverse of the following matrix (when it exists). The entries will involve \(x\).

\[ A = \begin{bmatrix}
x & 1 & 1 \\
1 & x & 1 \\
x & 2 & 1 \\
\end{bmatrix} \]

I would recommend Cramer’s rule (in our text or try the internet). The inverse can be written as the transpose of the matrix of cofactors (the \((i,j)\)-cofactor is \((-1)^{i+j} \det(M_{ij})\)) divided by the determinant.

4. Assume that \(A = MBM^{-1}\) for some invertible matrix \(M\). Show that \(\det(A - \lambda I) = \det(B - \lambda I)\). Distributive Law?

5. Let \(A\) be an \(n \times n\) matrix with an eigenvector \(v\) of eigenvalue \(t\). Assume we can obtain an invertible matrix \(M\) which has \(v\) as its first column. Compute the first column of \(B = M^{-1}AM\). Deduce that \(B\) has an eigenvalue \(t\).

6. Assume \(A\) is a \(3 \times 3\) matrix, and \(M\) is an invertible matrix with \(A = MDM^{-1}\), where \(D\) is the diagonal matrix

\[ D = \begin{bmatrix}
2 & 0 & 0 \\
0 & 3 & 0 \\
0 & 0 & 4 \\
\end{bmatrix} \]

Show that \((A - 2I)(A - 3I)(A - 4I) = 0\) where 0 denotes the \(3 \times 3\) matrix of 0’s.

7. Show that the \(3 \times 3\) matrix of 1’s is diagonalizable. Try to generalize to the \(n \times n\) matrix of 1’s.

8. Let \(A = (a_{ij})\) be an \(n \times n\) matrix with integral entries such that the diagonal entries are all not divisible by 3 (\(a_{ii}\) is not evenly divisible by 3) and all off diagonal entries are divisible by 3 (\(a_{ij}\) is divisible by 3 for \(i \neq j\)). Show that \(A\) has \(\det(A) \neq 0\), i.e. \(A\) is invertible.