1. Solve the differential equation given the initial conditions $x_1(0) = -3$, $x_2(0) = 4$.

\[
\frac{dx_1}{dt}(t) = \frac{dx_2}{dt}(t) = -2x_1(t) - 2x_2(t)
\]

The following should be helpful:

\[
\begin{bmatrix}
0 & 1 \\
-2 & -2
\end{bmatrix}
\begin{bmatrix}
-\frac{1}{2} - \frac{i}{2} \\
1
\end{bmatrix}
\begin{bmatrix}
-\frac{1}{2} + \frac{i}{2} \\
1
\end{bmatrix}
= \begin{bmatrix}
-1 + i & 0 \\
0 & -1 - i
\end{bmatrix}
\]

2. You are given a 3 dimensional vector space $V \subseteq \mathbb{R}^5$. Could there be a $3 \times 6$ matrix $A$ with nullspace of $A$ being $V$? Explain. Could there be $6 \times 5$ matrix $B$ with nullspace of $B$ being $V$? Explain. In either case, if you were given a basis for the three dimensional space $V$, how would you find the desired matrix assuming it exists.

3. Consider the two planes $\pi_1$: $x - y + 2z = 3$ and $\pi_2$: $x + 2y + 3z = 6$.
   a) Find the intersection of $\pi_1$ and $\pi_2$ in vector parametric form.
   b) What is the angle (or just the cosine of the angle) formed by the two planes?
   c) Find the distance of the point $(-1, 2, 2)$ to the plane $\pi_1$.
   d) Find the equation of the plane parallel to $\pi_1$ through the point $(3, -2, 0)$.
   e) Imagine the direction $(0, 0, 1)^T$ as pointing straight up from your current position $(0, 0, 0)^T$ in 3-space and the plane $\pi_2$ as a physical plane. If a marble is placed on $\pi_2$ at the point $(6, 0, 0)^T$, what direction will the marble roll under the influence of gravity?

4. Let $A$ and $B$ be similar matrices, namely there is an invertible matrix $M$ with $A = MBM^{-1}$. Show that $\text{tr}(A) = \text{tr}(B)$. Hint: There are various ways to see this. You could try showing that $\text{tr}(CD) = \text{tr}(DC)$ for square matrices.

5. Let $A$ be a $n \times n$ matrix of real entries satisfying $A^2 = -I$. Show that
   a) $A$ is invertible (or nonsingular)
   b) $A$ has no real eigenvalues
   c) $n$ is even
   d) (harder question) $\det(A) = 1$.

6. Consider two vectors spaces $U, V$, subspaces of $\mathbb{R}^m$. Define $U + V = \{u + v : u \in U, v \in V\}$. Show that $U + V$ is a vector space. Now show that $\dim(U) + \dim(V) = \dim(U \cap V) + \dim(U + V)$.
   (Hint: if we have an $m \times n$ matrix $A$ then $n = \dim(\text{nullspace}(A)) + \text{rank}(A)$. How should we form $A$?)

7. Consider the problem of 3D images with perspective. We are ‘projecting’ points in $\mathbb{R}^3$ onto a plane $\pi : z = 1$ where you consider your eye to be located at the origin (this is a typical problem in computer graphics). Given a point $(x, y, z)$ we say its image on $\pi$ is the intersection of the plane $\pi$ with the line joining the point $(x, y, z)$ and the origin. You might draw a picture to see what is going on. Now consider the image of a line $L : \{x + sd : s \in \mathbb{R}\}$ on $\pi$. What is its image (viewed as a sequence of points, one for each value of $s$) on the plane $\pi$. Give a description for how the image is traced out on $\pi$ as $s$ goes from 0 to $\infty$. 