Notation for Revised Simplex Formulas:

The standard inequality form of an LP is

$$\max\{\mathbf{c}\cdot\mathbf{x} \,:\, A\mathbf{x} \le \mathbf{b}, \quad \mathbf{x} \ge \mathbf{0}\}.$$

We add slack variables, one per inequality, to write this as

$$A\mathbf{x} + \mathbf{x}_S = [A\,I]\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_S\end{bmatrix} = \mathbf{b}, \qquad z = \mathbf{c}\cdot\mathbf{x} = [\mathbf{c}^T\mathbf{0}^T]\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_S\end{bmatrix},$$

where S denotes indices of the slack variables. The notation has been to use B to denote the basic variables and N to denote the non-basic variables. Thus $\mathbf{x}_B$ (respectively $\mathbf{c}_B$) denotes the vector of basic variables (the vector obtain from those rows of

$$\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_S\end{bmatrix} \quad \left(\text{respectively } \begin{bmatrix}\mathbf{c}\\ \mathbf{0}\end{bmatrix}\right)$$

indexed by B) and similarly $\mathbf{x}_N$ (and $\mathbf{c}_N$) is the vector indexed by N . We let A_N denote the submatrix of $[A\,I]$ of columns indexed by N and then let B (rather than the more consistent A_B) to denote the invertible submatrix of $[A\,I]$ of columns indexed by B. We can shuffle the indices of the variables as follows:

$$[A\,I]\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_S\end{bmatrix} = \sum_i A_i x_i = [A_N\,B]\begin{bmatrix}\mathbf{x}_N\\ \mathbf{x}_B\end{bmatrix} = A_N\mathbf{x}_N + B\mathbf{x}_B = \mathbf{b}.$$

We manipulate as follows

$$B\mathbf{x}_B = \mathbf{b} - A_N\mathbf{x}_N,$$

and then multiply on the left by B^{-1} to obtain

$$\mathbf{x}_B = B^{-1}\mathbf{b} - B^{-1}A_N\mathbf{x}_N.$$

We then take

$$z = \mathbf{c}\cdot\mathbf{x} = [\mathbf{c}^T\mathbf{0}^T]\begin{bmatrix}\mathbf{x}\\ \mathbf{x}_S\end{bmatrix} = \mathbf{c}_B^T\mathbf{x}_B + \mathbf{c}_N^T\mathbf{x}_N,$$

and substitute for $\mathbf{x}_B$ to obtain

$$z = \mathbf{c}_B^T B^{-1}\mathbf{b} + (\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1}A_N)\mathbf{x}_N.$$

Perhaps the only mystery is how B^{-1} is known to exist. I'll try that on an assignment 1.

Revised Simplex Formulas (please memorize!):

$$\mathbf{x}_B = B^{-1}\mathbf{b} - B^{-1}A_N\mathbf{x}_N$$
$$z = \mathbf{c}_B^T B^{-1}\mathbf{b} + (\mathbf{c}_N^T - \mathbf{c}_B^T B^{-1}A_N)\mathbf{x}_N$$