

Unbounded LP Example

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$$\begin{array}{rccccrc} \max & & 2x_2 & +x_3 & & & \\ & x_1 & -x_2 & +x_3 & \leq & 5 & \\ & -2x_1 & +x_2 & & \leq & 3 & \\ & & x_2 & -2x_3 & \leq & 5 & \\ x_4 & = & 5 & -x_1 & +x_2 & -x_3 & \\ x_5 & = & 3 & +2x_1 & -x_2 & & \\ x_6 & = & 5 & & -x_2 & +2x_3 & \\ z & = & & 2x_2 & +x_3 & & \end{array} \quad x_1, x_2, x_3 \geq 0$$

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$$x_4 = 5 \quad -x_1 \quad +x_2 \quad -x_3$$

$$x_5 = 3 \quad +2x_1 \quad -x_2$$

$$x_6 = 5 \quad \quad -x_2 \quad +2x_3$$

$$z = \quad \quad 2x_2 \quad +x_3$$

x_2 enters

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$$\begin{array}{rccccrc} \max & & 2x_2 & +x_3 & & & \\ & x_1 & -x_2 & +x_3 & \leq & 5 & \\ & -2x_1 & +x_2 & & \leq & 3 & \\ & & x_2 & -2x_3 & \leq & 5 & \end{array} \quad x_1, x_2, x_3 \geq 0$$

$$x_4 = 5 - x_1 + x_2 - x_3$$

$$x_5 = 3 + 2x_1 - x_2$$

$$x_6 = 5 - x_2 + 2x_3$$

$$z = 2x_2 + x_3$$

x_2 enters and x_5 leaves

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$$x_4 = 5 - x_1 + x_2 - x_3$$

$$x_5 = 3 + 2x_1 - x_2$$

$$x_6 = 5 - x_2 + 2x_3$$

$$z = 2x_2 + x_3$$

x_2 enters and x_5 leaves

$$x_4 = 8 + x_1 - x_5 - x_3$$

$$x_2 = 3 + 2x_1 - x_5$$

$$x_6 = 2 - 2x_1 + x_5 + 2x_3$$

$$z = 6 + 4x_1 - 2x_5 + x_3$$

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$$x_4 = 8 + x_1 - x_5 - x_3$$

$$x_2 = 3 + 2x_1 - x_5$$

$$x_6 = 2 - 2x_1 + x_5 + 2x_3$$

$$z = 6 + 4x_1 - 2x_5 + x_3$$

x_1 enters

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$$\begin{array}{rcccccl} \max & & 2x_2 & +x_3 & & \\ & x_1 & -x_2 & +x_3 & \leq & 5 \\ & -2x_1 & +x_2 & & \leq & 3 \\ & & x_2 & -2x_3 & \leq & 5 \end{array} \quad x_1, x_2, x_3 \geq 0$$

$$x_4 = 8 + x_1 - x_5 - x_3$$

$$x_2 = 3 + 2x_1 - x_5$$

$$x_6 = 2 - 2x_1 + x_5 + 2x_3$$

$$z = 6 + 4x_1 - 2x_5 + x_3$$

x_1 enters and x_6 leaves

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$$\begin{array}{rcll} \max & & 2x_2 & +x_3 \\ & x_1 & -x_2 & +x_3 \leq 5 \\ & -2x_1 & +x_2 & \leq 3 \\ & & x_2 & -2x_3 \leq 5 \end{array} \quad x_1, x_2, x_3 \geq 0$$

$$x_4 = 8 + x_1 - x_5 - x_3$$

$$x_2 = 3 + 2x_1 - x_5$$

$$x_6 = 2 - 2x_1 + x_5 + 2x_3$$

$$z = 6 + 4x_1 - 2x_5 + x_3$$

x_1 enters and x_6 leaves

$$x_4 = 9 - \frac{1}{2}x_6 - \frac{1}{2}x_5$$

$$x_2 = 5 - x_6 + 2x_3$$

$$x_1 = 1 - \frac{1}{2}x_6 + \frac{1}{2}x_5 + x_3$$

$$z = 10 - 2x_6 + 5x_3$$

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$$x_1, x_2, x_3 \geq 0$$

x_3 enters

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$$\begin{array}{rcccc} x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 \\ x_2 & = & 5 & -x_6 & +2x_3 \\ x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 \\ z & = & 10 & -2x_6 & & +5x_3 \end{array}$$

x_3 enters and no leaving variable (no restriction on increase to x_3)

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x_3 enters and no leaving variable (no restriction on increase to x_3)
Parametric solution showing that LP is unbounded:

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x_3 enters and no leaving variable (no restriction on increase to x_3)
Parametric solution showing that LP is unbounded:

$$\begin{array}{rcl} x_1 & = & 1 + t \\ x_2 & = & 5 + 2t \\ x_3 & = & t \\ x_4 & = & 9 \end{array}$$

with $x_5 = x_6 = 0$ and $z = 10 + 5t$ for $t \geq 0$.