## MATH 444: MATHEMATICAL RESEARCH AND WRITING

A possible project.
Determinants have been studied for many years and there are many important results. Two with amusing proofs are the following. First, the number of spanning trees of a graph can be computed as a determinant (relating to Laplace matrix of the graph). Second, the product rule for matrices $(\operatorname{det}(A B)=\operatorname{det}(A) \operatorname{det}(B))$ can be obtained using involutions with no need for the elementary matrices usually used for this purpose. A project would give some discussion of the definition(s) of determinant and their relevance. Other facts would include the ideas that for a matrix $A$ formed of vectors $\mathbf{v}_{1}, \mathbf{v}_{2}, \ldots, \mathbf{v}_{n}$, then $|\operatorname{det}(A)|$ would be the volume of the parallopiped formed by the $n$ vectors and the sign of $\operatorname{det}(A)$ would have something to say about the orientation of the vectors particularly in $\mathbf{R}^{2}$ and $\mathbf{R}^{3}$. There are other results/formulas such as the Cauchy-Binet formula that might deserve some coverage.

The project would be a small primer on determinants.

