MATH 444: MATHEMATICAL RESEARCH AND WRITING

A possible project.

Determinants have been studied for many years and there are many important results. Two with amusing proofs are the following. First, the number of spanning trees of a graph can be computed as a determinant (relating to Laplace matrix of the graph). Second, the product rule for matrices $(\det(AB) = \det(A) \det(B))$ can be obtained using involutions with no need for the elementary matrices usually used for this purpose. A project would give some discussion of the definition(s) of determinant and their relevance. Other facts would include the ideas that for a matrix A formed of vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$, then $|\det(A)|$ would be the volume of the parallopiped formed by the n vectors and the sign of $\det(A)$ would have something to say about the orientation of the vectors particularly in \mathbf{R}^2 and \mathbf{R}^3 . There are other results/formulas such as the Cauchy-Binet formula that might deserve some coverage.

The project would be a small primer on determinants.