A diffusion model for electricity prices

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Abstract

Starting from a simple supply/demand model for electricity, we obtain a diffusion (i.e. jumpless) model for spot prices which can exhibit 'price spikes'. We estimate the parameters in the model using historical data from the Alberta and California markets, and compare this model with some others used for spot prices.

Key words: electricity prices, diffusion, mean-reverting

1. Introduction

In recent years a number of electricity markets have been deregulated, and power is now a traded commodity. Most of these markets have a rather short history, and may still be evolving as market participants learn the features of these new markets. The behaviour of prices in these markets is poorly understood.

In this paper we will study two North American markets: Alberta, which deregulated prices on 1 January 1996, and California, which deregulated on 1 April 1998. The special features of power markets arise from the exceptional nature of electricity as a commodity. First, electricity is continuously generated and consumed, and in most areas there is no effective storage. Next, and rather paradoxically for a commodity which can be transported at very high speeds, the market is localized – capacity constraints on transmission lines (and the lack of storage) mean that power prices can vary very widely in different locations. For example, at 14.00 on 1 November 1999 the price for South Path 15 (SP15, the Los

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Angeles area of California) was \$45.00, while for NP15, the San Francisco region, it was \$103.79. Thirdly, prices show very strong daily effects – prices during the 16 'high load' hours (07.00-23.00 in California) are much higher than during the remaining 'low-load' hours. Power prices also have some other oddities: zero, and even negative prices can occur – though this is rare.

A more detailed analysis of prices shows a number of other features – a weekly effect (prices are usually slightly lower on Sundays), and also seasonal effects, which vary from place to place. Thus California is generally considered to be a 'summer peaking' market – demand, and prices, are highest in the summer due to air conditioning, while Alberta (with cold winters) has higher winter prices.

To avoid dealing with the daily effects, it is useful to consider spot power as 24 different commodities – one for each hour of the day. Figure 1 shows the average high load hour price for Alberta for the period 1 January 1996 – 23 August 2001.

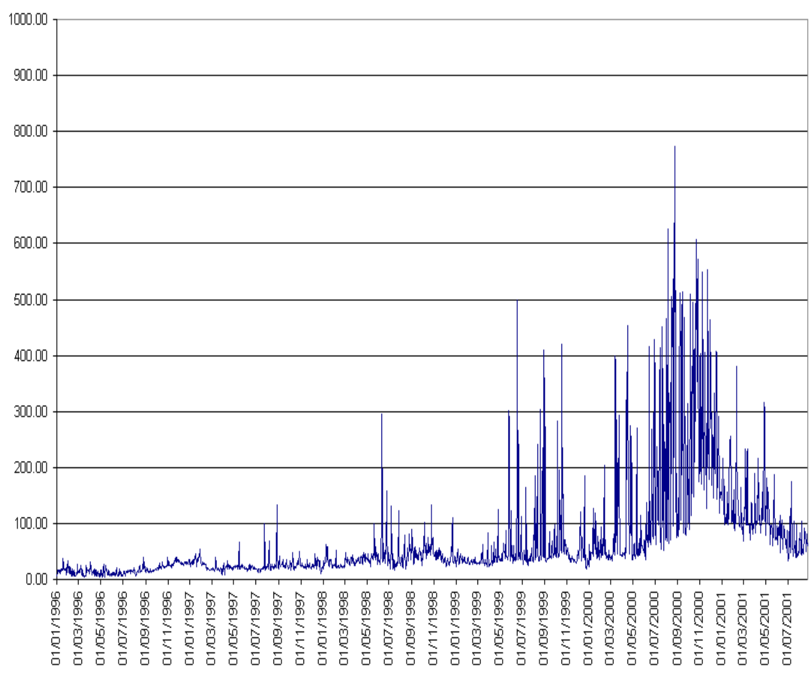

Figure 1. Alberta prices Jan 1996 – August 2001

The most dramatic feature of the graph is the presence of pronounced 'price spikes';

during these power prices can rise from their approximate average value of around C$ 40/MWhr to more than C$500. It is immediately clear from this graph that neither Brownian motion nor geometric Brownian motion is a reasonable model for the spot price.

In this paper, based on a simple microeconomic model inspired by Föllmer and Schweizer (1993), we give a (one-factor) diffusion model which can account for price spikes. Since the price arises from a model for each of the supply and demand curves for power, it is easy in principle to incorporate additional factors to account for long-term effects, or changes in market structure. Such extensions of our model would be essential before it could satisfactorily deal with options and futures prices. We also discuss the fit of this model with data from the Alberta and California markets.

The layout of the paper is as follows. In section 2 we discuss briefly some other models for power prices. In Section 3 we introduce our model, and discuss some possible extensions. In section 4 we estimate parameter values for the model for the two markets mentioned above.

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2. Models for power prices

Let S_t be spot price on day t. To avoid dealing at this point with daily effects, we mean by this either some average price on that day, for the high load hours for example, or the price at a specific time. We measure time in years, so we have a time series $\{S_t : t = 0, 1/365, \ldots, n/365\}$.

The usual geometric Brownian motion model for securities is

$$\begin{aligned} dX_t &= a dt + \sigma dW_t, \\ S_t &= e^{X_t}, \end{aligned} \tag{2.1}$$

where W is a standard one-dimensional Brownian motion. This model has no mean reverting behaviour, and completely fails to capture the behaviour of power prices.

Since power prices, like those of other energy commodities, do exhibit some mean-reverting behaviour, it is natural to look at models with this property. (See Schwartz (1997) for a discussion of why mean reversion for commodities is reasonable on general economic grounds). The simplest mean reverting diffusion is the Ornstein-Uhlenbeck $OU(\lambda, a, \sigma)$ process, defined by the stochastic differential equation (SDE)

$$\begin{aligned} dX_t &= -\lambda(X_t - a)dt + \sigma dW_t \\ S_t &= X_t. \end{aligned} \tag{2.2}$$

For this process λ^{-1} gives the time scale over which the process reverts to its mean a. This model has two serious difficulties. First, as we shall see below, it does not provide a good fit to the historical data. Also, it fails to give a satisfactory account of the relation between spot and forward prices. If $\mathcal{F}_t = \sigma(X_s : s \leq t)$ and we assume that the forward price at time t of spot power at time T is given by

$$F(t,T) = e^{(r-\mu)(T-t)} E(X_T | \mathcal{F}_t), \tag{2.3}$$

where r is the interest rate and μ is the convenience yield, then under model (2.2) we have

$$F(t,T) = e^{(r-\mu)(T-t)} \Big(a\big(1 - e^{-\lambda(T-t)}\big) + X_t e^{-\lambda(T-t)} \Big). \tag{2.4}$$

Estimates for λ from historical data tend to give estimates in the range 50-180, thus giving a characteristic time for mean reversion of 2-6 days. With a value for λ of this kind, even if $T - t$ is as little as one month, one has $e^{-\lambda(T-t)} < 0.007$, so that movements in the spot price X_t would have only a very small effect on forward prices. (More formally the volatility covariance $\langle X, F(.,T) \rangle$ is too small.) For the period when the California futures market was operating and liquid a much larger effect occurred.

In a paper on the Nordic power exchange, Lucia and Schwartz (2002), investigate the models

$$S_t = f(t) + X_t, \tag{2.5}$$

or

$$S_t = \exp(f(t) + X_t), \tag{2.6}$$

where X is $OU(0, \lambda, \sigma)$, and $f(t)$ is a deterministic function of t, which incorporates day of week, monthly and holiday effects. For this market, where price spikes are rare, they found that (2.5) gave the better fit to forward and future prices.

The usual way (see for example Schwartz (1997)) to handle the relation between spot and future prices is to introduce more 'factors'. Thus in (2.2) one could replace a by a stochastic process A_t, which might be a geometric Brownian motion, giving rise to a '2-factor mean reverting model': the actual price S_t performs short term oscillations around a long run 'mean' which, like standard securities, is modeled by a geometric Brownian motion. (See Dornier and Queruel (2000) for the need for care in the interpretation of this mean.)

Another model along these lines is the 'Pilipovic' model (see Pilipovic (1997))

$$\begin{aligned} dS_t &= \alpha(L_t - S_t)dt + S_t\sigma dW_t^{(1)}, \\ L_t &= \exp(\mu t + \sigma' W_t^{(2)}), \end{aligned} \tag{2.5}$$

where $W^{(i)}$ are independent Brownian motions.

None of the models above give rise to the 'price spikes' which are such a prominent feature of the actual spot prices in the Alberta and Californian markets. One possible approach to these is by including jump terms in these models. For example, a 2-factor mean reverting model involving jumps could take the form:

$$\begin{aligned} dX_t &= -\lambda(X_t - A_t)dt + \sigma dW_t^{(1)} + dN_t^{\nu}, \\ A_t &= \mu t + \sigma' W_t^{(2)}, \\ S_t &= e^{X_t}, \end{aligned} \tag{2.6}$$

where N^{ν} is a compound Poisson process with jump measure $\nu(dx)$.

Jump models of this kind involve several extra difficulties. Even the simplest require several extra parameters, for the jump rates and jump sizes in the jump measure ν. For standard securities, most jump models lead to an incomplete market, and while this will also be true for electricity, it is not such a serious disadvantage, since in any case spot electricity cannot be held. In addition, jump models such as (2.6) cannot easily account for price spikes. It is simple enough to obtain large upward jumps, by making the positive tail of the measure $\nu(dx)$ fat enough, but because the times of the large positive and negative jumps are independent Poisson processes, it is not possible to ensure that a large upward jump will be followed within a few days by a large downward jump.

One can allow the mean reversion term $-\lambda(X_t - A_t)dt$ to remove the spike – see for example Clewlow and Strickland (1999). However, this gives rise to quite large values of the mean reversion parameter λ – much larger than is found in fitting other models to the data.

One is therefore led to hidden Markov models, where the market can be in one of two states ('normal' or 'exceptional') and the price jumps when the hidden component switches states. For a sophisticated model along these lines see Elliott, Sick and Stein (2000), where an additional component Z_t, the number of power stations on line, is introduced. (We remark that while Z is a hidden component for an observer who only sees the price process S_t, this is not truly a hidden component since it will be known to market participants.)

It is very likely that, in the end, a realistic model for electricity prices will have to incorporate some jump components. (Even for standard securities jump models have many advocates.) However, because pure diffusion (i.e. jumpless) models are much simpler, it seems worthwhile to investigate these models first.

3. A supply/demand model for power prices

The following simple model for prices, based on stochastic models for the supply and demand curves, was inspired by the models in Föllmer and Schweizer (1993), and the Alberta Power Pool merit graph (see Figure 2).

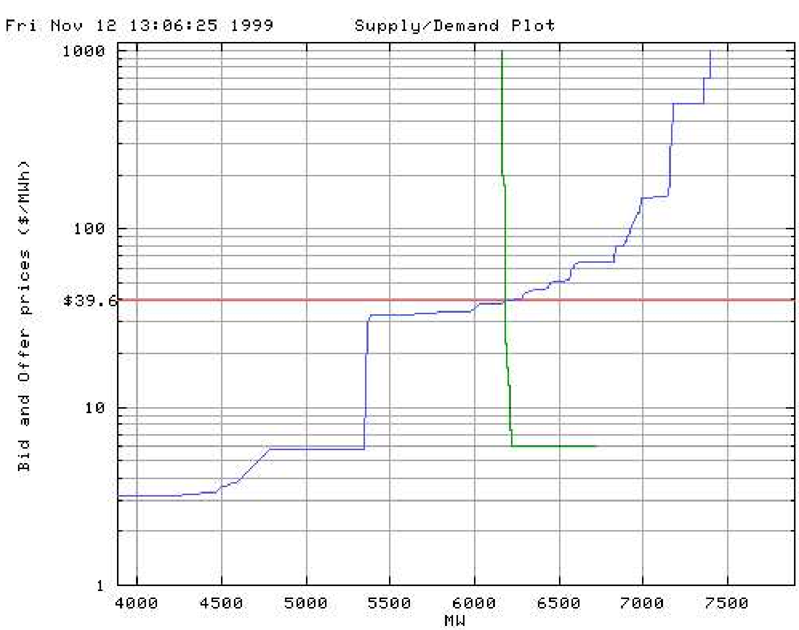

Figure 2. Alberta Power Pool merit graph for 13:06 on 12 November 1999.

Consider power at a single location, and let

$$u_t(x) = \text{ supply at time } t \text{ if the price is } \$x,$$

$$d_t(x) = \text{ demand at time } t \text{ if the price is } \$x.$$

(The graphs in Figure 2 actually show the inverse functions u_t^{-1}, d_t^{-1}.) We assume that $u_t(\cdot)$ is increasing, and $d_t(\cdot)$ is decreasing, and that the price at time t is the unique number S_t such that

$$u_t(S_t) = d_t(S_t). \tag{3.2}$$

We begin with quite simple forms for u_t and d_t. Assume that the supply is non-random and independent of t, so that

$$u_t(x) = g(x). \tag{3.3}$$

Further, demand for power is very inelastic, since in the current market structure few users pay spot prices. So we take

$$d_t(x) = D_t, \tag{3.4}$$

for some stochastic process D_t. Then $D_t = d_t(S_t) = u_t(S_t) = g(S_t)$, so that

$$S_t = g^{-1}(D_t). \tag{3.5}$$

In actual markets the total supply is limited, so we begin by considering g of the form:

$$g(x) = a_0 - b_0 x^{\alpha}, \tag{3.6}$$

where $\alpha < 0$. Thus

$$f(y) = g^{-1}(y) = \left(\frac{a_0 - y}{b_0}\right)^{1/\alpha}, \quad y < a_0. \tag{3.7}$$

If demand exceeds the maximum supply a_0 then S_t is capped at some maximum price. (This is quite reasonable on fundamental grounds – power markets do have a maximum price, which is sometimes reached.) So we obtain the model

$$S_t = \begin{cases} \left(\frac{a_0 - D_t}{b_0}\right)^{1/\alpha}, & D_t < a_0 - \varepsilon_0 b_0, \\ \varepsilon_0^{1/\alpha}, & D_t \geq a_0 - \varepsilon_0 b_0. \end{cases}$$

We model D_t by an $OU(\lambda, a_1, \sigma_1)$ process, and set

$$D_t = a_1 - \sigma_1 Y_t,$$

where Y is an $OU(\lambda, 0, 1)$ process. Then writing $F_t = \{D_t < a_0 - \varepsilon_0 b_0\} = \{Y_t > (a_1 - a_0 + b_0\varepsilon_0)/\sigma_1\}$, on F_t we have that

$$S_t = \Big(\frac{a_0 - a_1}{b_0} + \frac{\sigma_1 Y_t}{b_0}\Big)^{1/\alpha} = (1 + \alpha X_t)^{1/\alpha},$$

where

$$X_t = \frac{a_0 - a_1 - b_0}{\alpha b_0} + \frac{\sigma_1}{\alpha b_0} Y_t$$

is also an OU process.

Now define

$$f_\alpha(x) = (1 + \alpha x)^{1/\alpha}, \quad \alpha \neq 0, \qquad f_0(x) = e^x,$$

and let g_α be the inverse of f_α:

$$g_\alpha(x) = \frac{x^\alpha - 1}{\alpha}, \quad x > 0, \quad \alpha \neq 0, \qquad g_0(x) = \log x.$$

Thus, as long as the price S_t is below the maximum $A_0 = \varepsilon_0^{1/\alpha}$, we have that $g_\alpha(S_t)$ is an OU process.

The transformations g_α were introduced in Box and Cox (1964), and are used quite frequently in statistical applications. See the recent paper Chen, Lockhart and Stephens (2002) for a discussion and bibliography. In the present context the use of this transform is supported by the microeconomic arguments given above. Although we initially took $\alpha < 0$, the functions f_α and g_α are defined for any $\alpha \in \mathbb{R}$, and so we will not restrict α to $(-\infty, 0)$. Note also that f_α and g_α are both increasing, and that the domain of g_α is $(0, \infty)$ (or $[0, \infty)$ if $\alpha > 0$), while that for f_α is

$$(-\infty, -1/\alpha) \text{ if } \alpha < 0, \quad (-\infty, \infty) \text{ if } \alpha = 0, \quad (-1/\alpha, \infty) \text{ if } \alpha > 0.$$

We therefore obtain the price model;

$$\begin{aligned} S_t &= \begin{cases} f_\alpha(X_t), & 1 + \alpha X_t > \varepsilon_0, \\ \varepsilon_0^{1/\alpha}, & 1 + \alpha X_t \leq \varepsilon_0, \end{cases} \\ dX_t &= -\lambda(X_t - a)dt + \sigma dW_t. \end{aligned} \tag{3.8}$$

We call this a 'non-linear Ornstein-Uhlenbeck' or $NLOU(\lambda, a, \sigma, \alpha, \varepsilon_0)$ process. Of the parameters, the maximum price $A_0 = \varepsilon_0^{1/\alpha}$ is a known constant for each particular market, and will usually be omitted. The remaining parameters λ a, σ and α have to be estimated – see Section 4. Note that f_α grows more slowly than an exponential if $\alpha > 0$, and more rapidly if $\alpha < 0$. In general terms, the smaller the value of α, the more pronounced the price spikes in the NLOU process will tend to be.

A plot of a simulation of an NLOU process is given in Figure 3 below.

Remarks. 1. If $\alpha = 0$ then $S_t = \exp(X_t)$, where X is an OU process, while if $\alpha = 1$ then S is an OU process. So, since we have embedded the models considered in Lucia and Schwartz (2002) in a parametric family, we can test these possibilities against general alternatives.

2. If $\alpha > 0$ then $\varepsilon_0^{1/\alpha}$ represents a minimum, rather than a maximum price. It will be convenient in this case just to take $\varepsilon_0 = 0$.

3. For the remainder of this paper we will examine the basic model given by (3.8). But it is worth remarking on some modifications that can (and should) be made in the direction of greater realism.

First, one might wish to replace the smooth functions g and $f = g^{-1}$ by functions which follow the supply curve more accurately. Figure 2 shows that the graph of g^{-1} (i.e. the function from supply to price), has many intervals of constancy, separated by quite short intervals on which the price rises rapidly. This is reflected by actual market behaviour – it is quite common to see a period of several hours over which the price is constant followed by a jump.

Next, one could incorporate seasonal and daily effects into the demand process, for example by replacing D_t by $D_t + \varphi(t)$ where

$$\varphi(t) = a_1 \cos\left(2\pi(t - t_0)\right) + a_2 \cos\left((2\pi \times 365)(t - t_1)\right);$$

here a_1 measures the strength of the yearly oscillation, and a_2 the daily oscillation. (It may also be necessary to incorporate a weekly factor, as well as special terms for public holidays.)

One can also replace the constants a_0 and b_0 in (3.6) with stochastic processes A_t, B_t, or deterministic functions of time, which would describe various kinds of changes to the supply function. This could be either longer term economic effects, such as variation in fuel prices, or from short term events. For example, a simple model which incorporates the effects of power outages would be to take A_t as a 2-state Markov chain with values a or a'.

4. Maximum Likelihood Estimation and Data Analysis

It is is not hard to calculate the MLE for the $NLOU(\lambda, a, \sigma, \alpha)$ process. Let $\Delta = 1/365$, so that we observe $\widehat{S}_k = S_{k\Delta}$, $0 \le k \le n$. Then (neglecting the cutoff effects), since $g_\alpha(S_t)$ is an $OU(\lambda, a, \sigma)$ process, we have that $Y_k = g_\alpha(S_{k\Delta})$, is an $AR(1)$ process given by

$$Y_k = b + \rho Y_{k-1} + \theta^{1/2}\eta_k, \qquad 0 \le k \le n. \tag{4.1}$$

Here η_k are i.i.d. $N(0,1)$ r.v., and

$$\rho = e^{-\lambda\Delta}, \qquad b = a(1 - e^{-\lambda\Delta}), \qquad \theta = \frac{\sigma^2(1 - e^{-2\lambda\Delta})}{2\lambda}. \tag{4.2}$$

Note that if $\lambda\Delta \ll 1$ then the forward Euler approximation to the SDE (2.2) is fairly accurate, and

$$\rho \simeq 1 - \lambda\Delta, \quad b \simeq a\lambda\Delta, \quad \theta \simeq \Delta\sigma^2, \tag{4.3}$$

with errors of order $O((\lambda\Delta)^2)$. This would be quite accurate for many commodities, where mean reversion times are typically of the order of a few years, so that $\lambda\Delta < 0.01$. However, for the markets we consider, we typically have $\lambda\Delta$ in the range 0.2–0.5, so that (4.3) may be a poor approximation.

The transition density $q(x,y)$ of Y is given by

$$q(x,y) = (2\pi\theta)^{-\frac{1}{2}} e^{-(y - x\rho - b)^2/2\theta}, \tag{4.4}$$

and so that of $\widehat{S}$ is

$$\begin{aligned} p(x,y) &= q(g_\alpha(x), g_\alpha(y))|g'_\alpha(y)| \\ &= |g'_\alpha(y)|(2\pi\theta)^{-\frac{1}{2}} \exp\Big(-\frac{1}{2\theta}\big(g_\alpha(y) - g_\alpha(x)\rho - b\big)^2\Big). \end{aligned} \tag{4.5}$$

We look at the conditional likelihood for $\widehat{S}$, $L(\widehat{S}_1, \dots, \widehat{S}_n | \widehat{S}_0) = \prod_{i=1}^n p(\widehat{S}_{i-1}, \widehat{S}_i)$, which has log-likelihood $\widetilde{L}$ given by

$$\widetilde{L} = \sum_{i=1}^n \log|g'_\alpha(\widehat{S}_i)| - \frac{n}{2}\log(2\pi\theta) - \frac{1}{2\theta}\sum_{i=1}^n \big(g_\alpha(\widehat{S}_i) - \rho g_\alpha(\widehat{S}_{i-1}) - b\big)^2. \tag{4.6}$$

For a fixed α, it is routine to minimize this over b, ρ and θ to obtain the log-likelihood given α:

$$\widetilde{L}(\widehat{S}|\alpha) = c_n + \sum_{i=1}^{n} \log|g'_\alpha(U_i)| - \frac{n}{2}\log\widehat{\theta}. \tag{4.7}$$

This can then be minimized numerically to obtain the MLE for α. Error estimates were calculated using the usual second derivative method.

Remarks. 1. For some values of α a little care is required in the numerical computation of g_α. We usually have $10 < x < 1000$. When α is close to zero x^α is close to 1, and standard floating point operations give an inaccurate value for $g_\alpha(x) = (x^\alpha - 1)/\alpha$. So, for small α it is better to use the approximations $g_\alpha(x) = \log x + \frac{1}{2}\alpha(\log x)^2 + O(\alpha^2)$. Also, if $\alpha \leq -2$ and $x > 100$, one has $x^\alpha \ll 1$, and again values of g_α can be inaccurate. It is better to first calculate the MLEs associated with $\widetilde{g}_\alpha(x) = x^\alpha = \alpha g_\alpha(x) + 1$, and then transform.

2. The procedure above will work provided the prices $S_{k\Delta}$ are below the cutoff value A_0 of the maximum price. If the series $S_{k\Delta}, 0 \leq k \leq n$ contained a significant number of prices at A_0 then $Y_k = g_\alpha(S_{k\Delta})$ will no longer be an $AR(1)$ process. In this case one would need to consider MLEs for a truncated process of the form $\widetilde{Y}_k = \max(Y_k, y_0), \quad k = 0, \ldots, n$.

Data Analysis

The model was tested on data from the Alberta and California markets.

Alberta was the first region in North America to deregulate electricity, and hourly prices were available for the period 1 January 1996 – 23 August 2001, a period of 2062 days. The market is relatively small, and isolated, with limited connections in terms of transmission capacity with the neighboring Canadian provinces of Saskatchewan and British Columbia. We studied the series of daily average prices at high load hours. The series shows a clear increase over the period, with wildly oscillating prices in the second half of 2000, when electricity markets in North Western North America experienced shortages and exceptionally high prices.

We therefore split the series into three periods:

Period I	1 Jan 1996 – 9 Mar 1998	(800 days)

Period II	10 Mar 1998 – 18 May 2000	(800 days)
Period III	19 May 2000 – 23 Aug 2001	(462 days)

The values and error estimates for the parameters λ, a, σ and α are given in Table 1.

Alberta data	λ	a	σ	α	χ^2
Period I	98.0 ± 10.8	4.05 ± 0.27	11.36 ± 1.32	0.19 ± 0.04	24.7
Period II	172.1 ± 16.2	0.91 ± 0.04	0.12 ± 0.03	-1.08 ± 0.06	469.5
Period III	104.5 ± 14.6	2.32 ± 0.26	1.71 ± 0.52	-0.35 ± 0.06	32.1

Table 1: Alberta data and parameter estimates

The final column shows the χ^2 statistic for a likelihood ratio test of $H_0 : \alpha = 0$ vs H_1 $\alpha \in \mathbb{R}$; all are quite strongly significant. (The hypothesis that $\alpha = 1$, so that the price is an OU process given by (2.2), is even more strongly rejected.)

Figure 3 shows a simulation of a NLOU process with the parameters for Alberta, Period II.

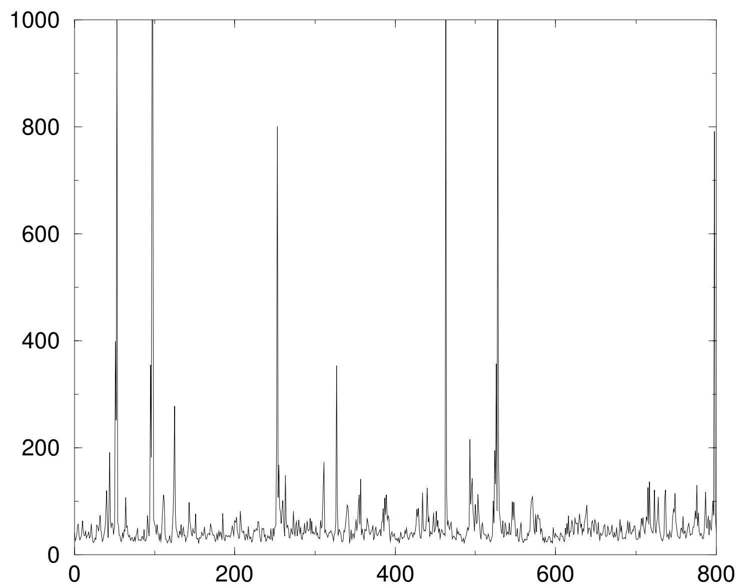

Figure 3. NLOU Process fitted to Alberta data for Period II.

To examine the effect of using the NLOU model, rather than a simple OU (i.e. $\alpha = 1$) or the Schwartz model ($\alpha = 0$), we can also fit these models to the data. For simplicity, we just do this for Alberta, Period II. Table 2 shows that this leads to much higher values of the mean reversion parameter λ.

α	λ	a	σ
0	204.1	3.82	10.83
1	306.5	55.3	1396.5

Table 2: Parameter estimates for Alberta Period II for fixed α.

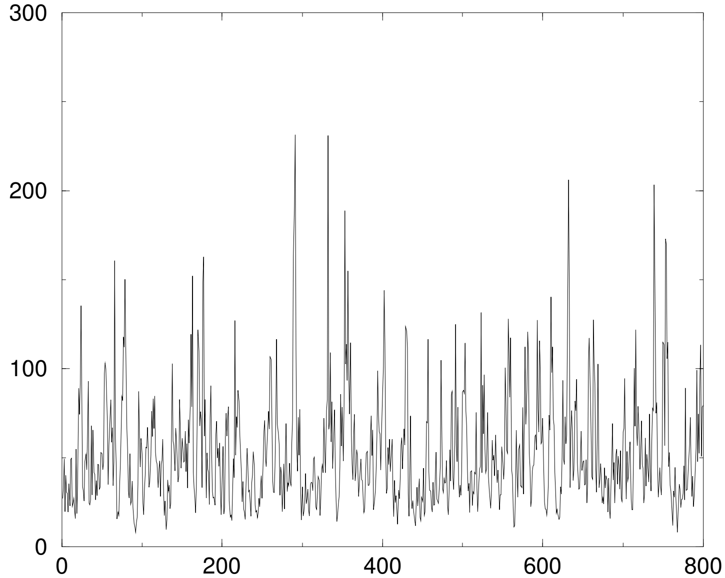

Figure 4. Exponential of OU Process fitted to Alberta data for Period II.

Figure 4 shows a simulation of e^{X_t} with $X \sim OU(204.1, 3.82, 10.83)$. Comparing Figure 1 with Figures 3 and 4 one sees that the exponential model ($\alpha = 0$) gives rise to a series with fewer spikes than the true data, while the fitted NLOU model with $\alpha = -1.08$ may actually produce too many spikes.

The California market was run by the California Power Exchange, which began posting prices on 1 April 1998. In the autumn of 2000, very high prices and capacity constraints caused increasing difficulties, and after 31 January 2001 the power exchange stopped posting prices. The initial few months of the California exchange show anomalously low prices: it is reasonable to assume that this was due to a period of learning by market participants, and exclude this from the analysis. So, as for Alberta, the overall series (1 April 1998 – 31 January 2001) was divided into three periods:

Period I	1 Apr 1998 – 14 Jun 1998	(75 days)
Period II	15 Jun 1998 – 14 Jun 2000	(731 days)
Period III	15 Jun 2000 – 31 Jan 2000	(431 days)

Unlike Alberta, which is a single unified market, California has prices associated with many different nodes or regions. We looked at prices in two of these: North Path 15 (NP15, northern California) and South Path 15 (SP15, southern California). The values and error estimates for the parameters λ, a, σ and α are given in Table 3. It is interesting that the estimates for α do not vary much over these different periods

California data	λ	a	σ	α	χ^2
NP-Period II	66.0 ± 9.3	1.97 ± 0.11	1.24 ± 0.17	-0.36 ± 0.04	99.2
NP-Period III	72.2 ± 16.6	2.36 ± 0.34	1.22 ± 0.50	-0.35 ± 0.08	19.4
SP-Period II	104.9 ± 12.4	2.20 ± 0.14	2.06 ± 0.31	-0.29 ± 0.04	44.9
SP-Period III	109.3 ± 22.0	2.86 ± 0.48	2.58 ± 1.10	-0.25 ± 0.08	8.5

Table 3: California data

The final column shows the χ^2 statistic for a likelihood ratio test of $H_0 : \alpha = 0$ vs H_1 $\alpha \in \mathbb{R}$; all are very strongly significant except the final one (South Path, period III), which is still significant at the 99% level.

A defect of this data analysis is that we are fitting a stationary model to series which (fairly plainly) are not stationary. We have controlled the problem to some extent by breaking the series into periods, but a more realistic approach would be to consider some of the extensions to the original model mentioned at the end of Section 3.

5. Concluding Remarks

In this paper we introduced a 'non-linear Ornstein-Uhlenbeck' (NLOU) model for spot power prices, which contains as special cases the simple OU model and also the Schwartz model. This model is jumpless (i.e. pure diffusion) but produces prices series which do have spikes. Fitting to data from Alberta and California indicates that this model does provide a better fit than either of the models mentioned above. (See the remarks on the power stack function in Eydeland and Geman (1999) for a similar conclusion.)

One disadvantage of this model is that it is unlikely that closed form expressions for expectations such as $E(S_T|\mathcal{F}_t)$ will exist for general α.

In addition, this model is stationary, and so will not provide a satisfactory explanation of the relation between spot and future prices. To do this, it seems necessary to look at multi-factor models, along the lines indicated at the end of section 3. Similar extensions would also be needed to use this model for option pricing.

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