Correction to: Loop Erased Walks and Uniform Spanning Trees.

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Theorem 5.2, which is stated as being 'implicit in [BJKS, KM08], is actually false. The bounds on the resistances $R_{\text{eff}}(o,x)$ are not enough to give control of the quantity that is needed, which is $R_{\text{eff}}(o,B(o,R)^c)$. Fortunately Theorem 5.2 was given for illustration only; the result which is used in the paper is Theorem 5.3, which *is* proved in [BJKS, KM08].

For Theorem 5.1 the control on $R_{\text{eff}}(o,B(o,R)^c)$ is obtained from the point to point resistances by using estimates on $V(x,r)$ and $R_{\text{eff}}(x,y)$ for base points x other than just o.

To see that Theorem 5.2 is actually false, rather than just not proved, consider the following example. Let G be a 'comb graph' of the following type. We take $G=(V,E)$ where $V=\mathbb{Z}_+^2$ and the the edges are of the form

$$E=\big\{\{(k,0),(k+1,0),k\geq 0\}\big\}\cup\big\{\{(k,j),(k,j+1),k\geq 0,j\geq 0\}\big\}.$$

Then G is a tree, and if $o=(0,0)$ we have $V(o,r)\asymp r^2$, so this graph satisfies the conditions of Theorem 5.2 with $\alpha=2$.

Let $r\geq 1$, and $k=r^{1/2}$. Consider the flow I from o to $B(o,r)^c$ obtained by making a flow of $1/k$ upwards in each of the first k teeth. This flow has energy $E(I)$ bounded by

$$\sum_{i=0}^{k}(r-i)k^{-2}+\sum_{i=0}^{k}(1-i/k)\leq rk^{-1}+k\leq 2r^{1/2}.$$

Thus $R_{\text{eff}}(o,B(o,r)^c)\leq cr^{1/2}$.

Hence, writing $B=B(o,r)$,

$$\mathbb{E}^o\tau(o,r)\leq\sum_{x\in B}g_B(o,x)\leq g_B(o,o)V(o,r)\leq cr^{5/2}=cr^{\alpha+\frac{1}{2}}.$$

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References

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