Mathematics 331 – Homework #2

due Friday, January 28 at the beginning of class

Reading: Sections 2.1 to 2.3

Warmup questions (don't hand in): 2-1, 2-2, 2-4

Homework (due January 28):

1. 2-6 from the text.
2. Let $F_k(x) = \sum_n \left\{ {n \atop k} \right\} \frac{x^n}{n!}$ be the exponential generating function for the Stirling numbers of the second kind.
   (a) Show that $F_k'(x) = kF_k(x) + F_{k-1}(x)$ for $k \geq 1$.
   (b) Use this to prove that $F_k(x) = \frac{(e^x - 1)^k}{k!}$.
3. Use (exponential) generating functions to prove the identity
$$\left\{ {n+1 \atop m+1} \right\} = \sum_k \binom{n}{k} \left\{ {k \atop m} \right\}.$$
4. Let F_n denote the nth Fibonacci number. Use generating functions to prove the identity
$$\sum_{k=0}^{n} F_k F_{n-k} = \frac{2nF_{n+1} - (n+1)F_n}{5}.$$
5. Find a closed form for the generating function $\sum_n F_n^2 x^n$ of the squares of the Fibonacci numbers.
6. 2-27 from the text.
7. 2-23 from the text.