Mathematics 331 – Homework #3

due Friday, February 11 at the beginning of class

Reminder: There will be no class on Monday, January 31. There will be a guest lecturer, Professor Greg Martin, on Wednesday, February 2 and Friday, February 4.

Reading: Section 2.6

Warmup questions (don't hand in): 2-15, 2-16, 2-17, 2-18, 2-28

Homework (due February 11):

1. 2–30(b)(c) from the text.
2. 2–29(c)(d)(e)(f)(g) from the text.
3. Let $g(n)$ be the function that satisfies the equation $\sum_{d|n} g(d)\phi(n/d) = 1$, for all positive integers n. Find the Dirichlet series generating function for g and deduce an explicit formula for g.
4. Let $\kappa(n)$ be defined by $\kappa(1) = 1$ and $\kappa(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = \alpha_1\alpha_2\cdots\alpha_k$. Show that
$$\sum_{n=1}^{\infty} \frac{\kappa(n)}{n^s} = \frac{\zeta(s)\zeta(2s)\zeta(3s)}{\zeta(6s)}.$$
5. Let $\nu(1) = 0$ and $\nu(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}) = k$ (i.e. $\nu(n)$ is the number of distinct prime factors of n). Show that
$$\sum_{n=1}^{\infty} \frac{\nu(n)}{n^s} = \zeta(s) \sum_{p} \frac{1}{p^s}.$$
6. Let $d(n)$ denote the number of positive divisors of n. Show that
$$\sum_{n=1}^{\infty} \frac{d(n^2)}{n^s} = \frac{\zeta^3(s)}{\zeta(2s)}.$$
7. 2–34 from the text.