Mathematics 440 & 508

Homework #8

Extra Problem: Show that if $z = g(w)$ is the solution of $w = z(1+z^2)$ that takes on the value 0 at $w = 0$, then

$$\tan^{-1} g(w) = w - \frac{4}{1!}\frac{w^3}{3} + \frac{6\cdot 7}{2!}\frac{w^5}{5} - \frac{8\cdot 9\cdot 10}{3!}\frac{w^7}{7} + \dots \quad \text{for} \quad |w| < 2\sqrt{3}/9.$$

Hint: First consider the integral

$$\frac{1}{2\pi i}\int_\gamma \frac{h'(z)}{f(z)-w}\,dz,$$

where $f(z) = z(1+z^2)$, $h(z)$ is the principal branch of $\tan^{-1} z$, and γ is a small circle around 0.