[10] 1. Use the Basic Formula to find the general solution of the differential equation

$$y' + y = x \sin x$$

and then find the solution satisfying the initial condition $y(0) = 2$.

[10] 2. Using Runge-Kutta of order 4 to solve the differential equation & initial conditions

$$y' = x + y - y^3, \quad y(0) = 1,$$

with step size $h = 0.50$ I obtained the estimate $y(1) = 1.2492310146$, while with $h = 0.25$ I obtained the estimate $y(1) = 1.2476075360$.

(a) What is the best estimate you can make for $y(1)$ from this data?

(b) What step size should I have chosen to get 10 decimal accuracy for the calculated $y(1)$?

[15] 3. Water flows into a deep cylindrical tank of cross-sectional area 1 square meter at the rate of 2 litres per second, and flows out through a small hole at the bottom. The hole is rigged so that the flow out increases with height, and is given exactly as $1.6h$ litres per second, where h is given in metres.

(a) If the initial height is 50 cm, find the exact formula for h as a function of t.

(b) Describe the eventual behaviour of the water in the tank.

[15] 4. Do two steps of Euler's method to find an approximation to $y(0.2)$, where $y(t)$ is the solution of the differential equation

$$y'' + y - y^3 = 1$$

such that $y(0) = 0$, $y'(0) = 1$.

The University of British Columbia
Midterm Examination
October 15, 1993

Mathematics 215
Section 102
Instructor: Bill Casselman

Duration: 50 minutes

- (•) Be sure that you have 4 pages in addition to this one.
- (•) Put your name below and on the **back** of the other pages. I like to grade 'blind' but your paper needs to be identified if it separates from the others.
- (•) In all questions, **you must show work** — i.e. display intermediate results — to get full credit.
- (•) You may not use notes of any kind.
- (•) Be neat! I will not attempt to decipher messy calculations.
- (•) Use complex exponentials where possible.
- (•) *All work you wish to be graded must be placed on these sheets. Work books should be used for scrap work only, and must not be handed in. I will simply throw away any other papers I receive.*

First Name _____ **Last Name** _____

Signature _____ **Student Number** _____

Rules governing examinations

- Each candidate should be prepared to produce upon request his library/AMS card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  - (a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
  - (b) Speaking or communicating with other candidates.
  - (c) Purposely exposing written papers to the view of other candidates.
- Smoking is not permitted during examinations.

1.	2.	3.	4.	Total
10	10	15	15	50